Rational Decision Making

Actual Uses for Decision Theory

- Kidney abnormality: Cyst or Tumor
  - Cyst test: aspiration
    - Needle in back to kidney- local anesthetic, in and out.
  - Tumor test: arteriography
    - Tube up leg artery to kidney, biopsy cut from kidney. 2 days in hospital. Lots of pain, risk of blood clot 10 times as great.
  - Patients preferred aspiration test (At), and found it 10 times better than Tumor test (Tt)
  - Utility theory says: U(At) = -1. U(Tt) = -10
  - At first then Tt

    $E[U_{AtTt}] = -1\,(1 - p(\text{Tumor})) + -11\,(p(\text{Tumor}))$
  - Tt first then At

    $E[U_{TtAt}] = -10\,p(\text{Tumor}) + -11\,(1 - p(\text{Tumor}))$

    Combining, $E[U_{AtTt}] > E[U_{TtAt}]$ when $p(\text{Tumor}) < 10/11$

Tt actually performed when doctors judged $p(\text{Tumor}) > 1/2$

Decision Theoretic Approaches to Problems in Cognition

- Analysis:
  - Goals of cognitive system, Environment model, Optimal strategy to accomplish goals
- Memory: Forget or Forget me not?
  - Goal: Store relevant information and allow efficient retrieval
    - Utility function: Assign utility for recall and memory search.
    - Relevant state: Need data or Not need data--Binary need variable.
    - Environment- Supplies event frequency of symbols for recall: Compute Belief about need.
  - Forgetting Strategy:
    - Risk = $P(N_{eed}=1)U(\text{Retrival}|\ N_{eed}=1)$
      $+ P(N_{eed}=0)U(\text{Retrival}|\ N_{eed}=0)$

Utility table	$N_{eed}=1$	$N_{eed}=0$
$U(R=1 \mid N_{eed})$	G - C	C
$U(R=0 \mid N_{eed})$	-G	0

Do the Math

$$
\begin{aligned}
Value(R=1) &= p_N U(R=1 \mid N) + (1-p_N)U(R=1 \mid \sim N) \\
&= p_N(G-C) + (1-p_N)(-C) \\
Value(R=0) &= p_N U(R=0 \mid N) + (1-p_N)U(R=0 \mid \sim N) \\
&= p_N(-G) + (1-p_N)(0)
\end{aligned}
$$

$$
\begin{aligned}
&Value(R=0) > Value(R=1)\,? \\
&p_N(-G) + (1-p_N)(0) > p_N(G-C) + (1-p_N)(-C) \\
&C/2 > p_N G
\end{aligned}
$$

Thus forgetting should be determined by the need probability

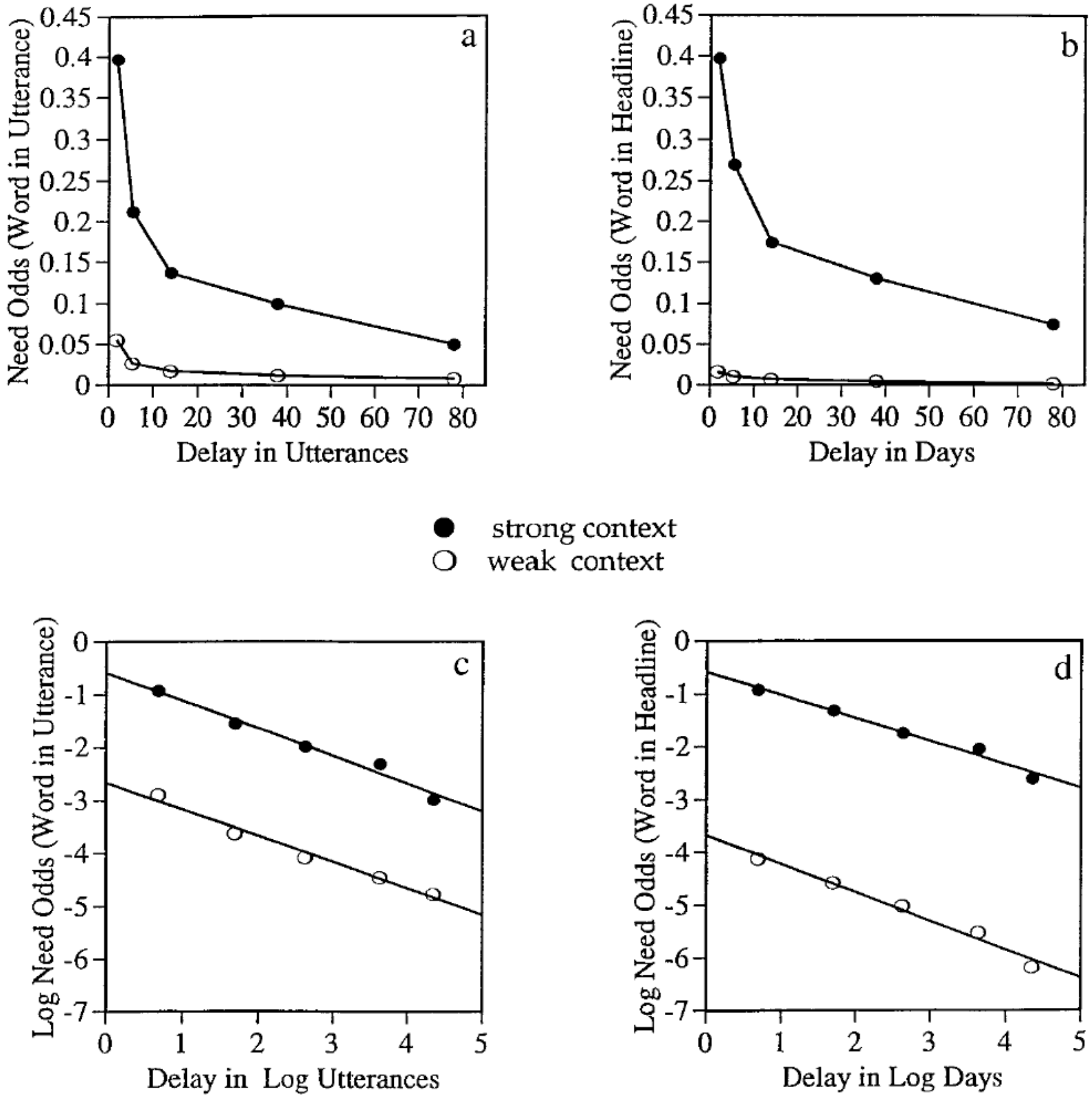


FIG. 5. Environmental recency curves from the analysis of the CHILDES and New York Times database. The left panels show the odds of a word being mentioned in an utterance as a function of the number of intervening utterances since it was last mentioned and whether the utterance included a strong associate (strong context) or did not (weak context). The right panels show the odds of a word being included in a particular headline as a function of the number of days since the word was last included and whether the headline included a strong associate. Parallel lines in (c) and (d) are consistent with environmental predictions of the rational analysis. The results are summarized in Table 2.

Assume that an efficient memory system is one where the availability of a memory structure, S, is directly related to the probability that it will be needed.

Empirically,

P(need) = at^{-k}

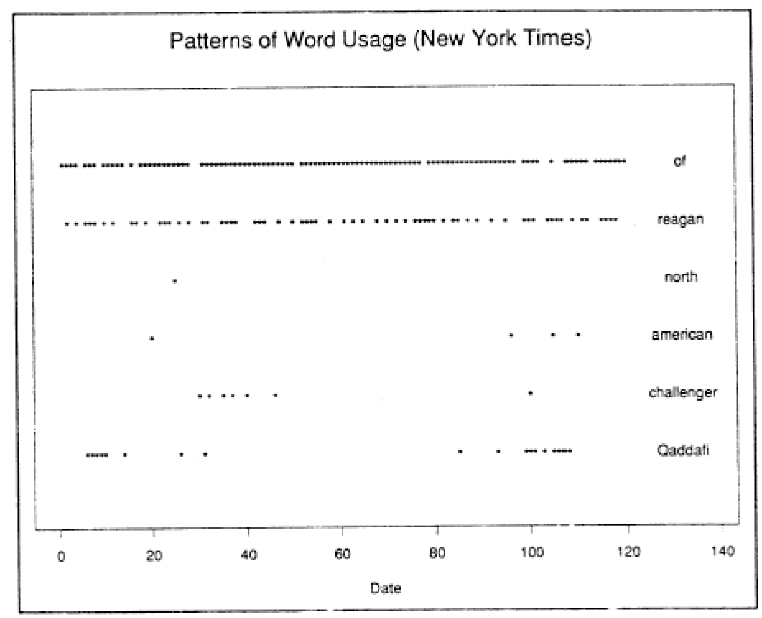
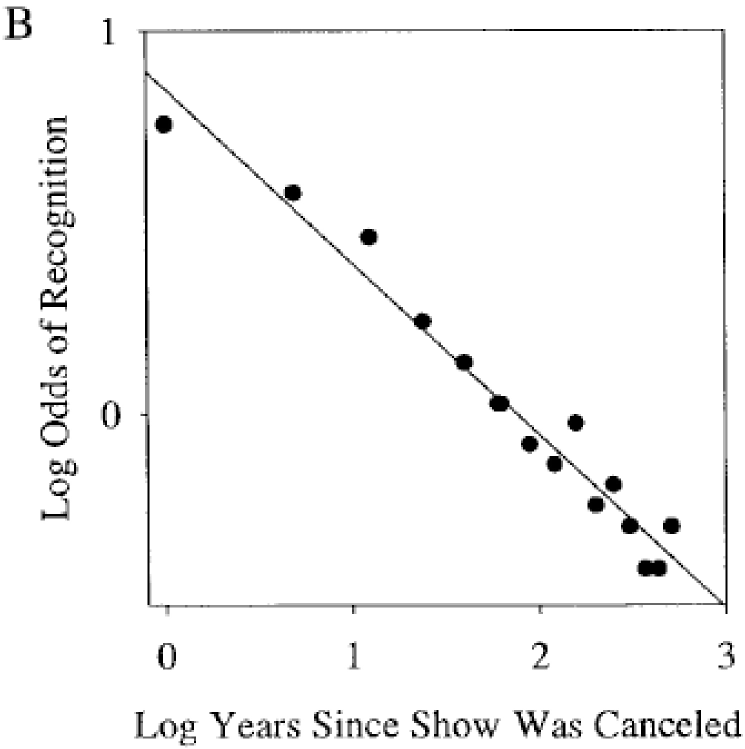
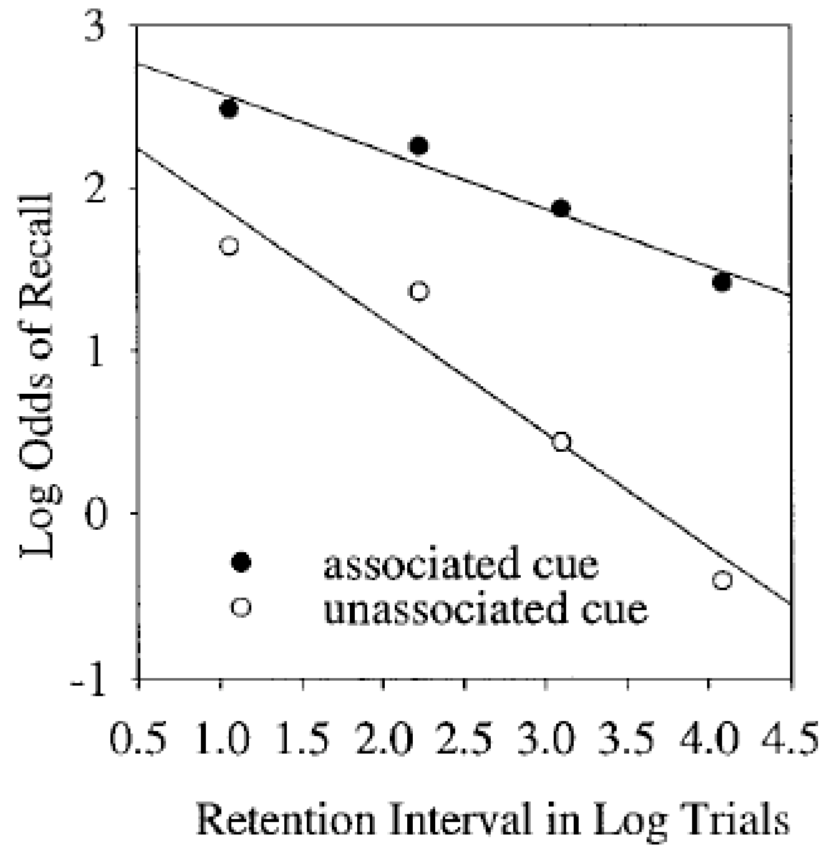


Fig. 5. Patterns of usage of various words in the *New York Times* data base over a 100-day period.



Recognition for television shows. Retention function from Squire (1989), adjusted for guessing, in log–log coordinates.



Subjects studied words and later recalled them after various retention intervals and in the presence of cues (other words) that were either strongly associated or unassociated to the target word.

Failures of Decision Theory as Model of Human Judgment

Allais (1953) Paradox (Certainty effect)

- A: Receive $1 million with p = 1.0

  E[$] = $1m
- B: (p=.1, $2.5 million), (p=.89,$1 million), (p=0.01, $0.0)

  E[$] = $1.14m

Utility analysis

$$U(\$1m) > .1\, U(\$2.5m) + .89\, U(\$1m) + .01\, U(\$0)$$

Let $U(\$0) = 0$

$$.11\, U(\$1m) > .1\, U(\$2.5m)$$

So let's do the implied Gamble:

- A: Receive $1 million with p = .11, else nothing
- B: Receive $2.5 million with p = .10, else nothing

Ellsberg Paradox

- Prefer
- I to II
- IV to III

BALLS IN URN

		30 Red	60 Black	60 Yellow
CHOICE	I	$100	nothing	nothing
	II	nothing	$100	nothing
CHOICE	III	$100	nothing	$100
	IV	nothing	$100	$100

FIGURE 12.10. The Gambles Presented in Ellsberg's Paradox

Violates independence of alternatives

Violations of Decision Theory

Framing Effects: Description invariance. Equivalent scenarios should result in same preferences, but do not.

Nonlinear preferences: Utility of a risky gamble should be linear in the probabilities.

Source dependence: Willingness to bet on uncertain event depends on the source rather than only the uncertainty. (Rather bet in area of competence with uncertain probabilities than a matched chance event (Heath & Tversky, 1991)

Risk Seeking: People sometimes do not minimize risk. (Sure loss vs. prob of a larger loss.

Loss Aversion: Losses loom larger than gains.

Describing Human Judgement

- Prospect Theory (Kahneman & Tversky)
  - Generalized decision theory
    - Replace probabilities with Weights w_i
    - Replace utilities by values v_i
    - Decide by computing the Overall value
    - $V = \sum_i w_i(p_i)\, v_i$

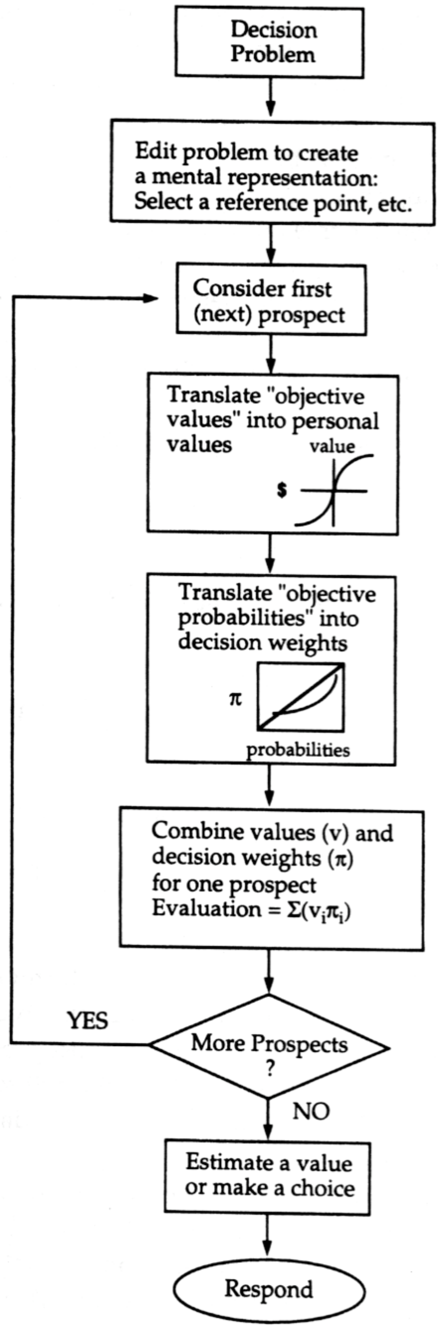
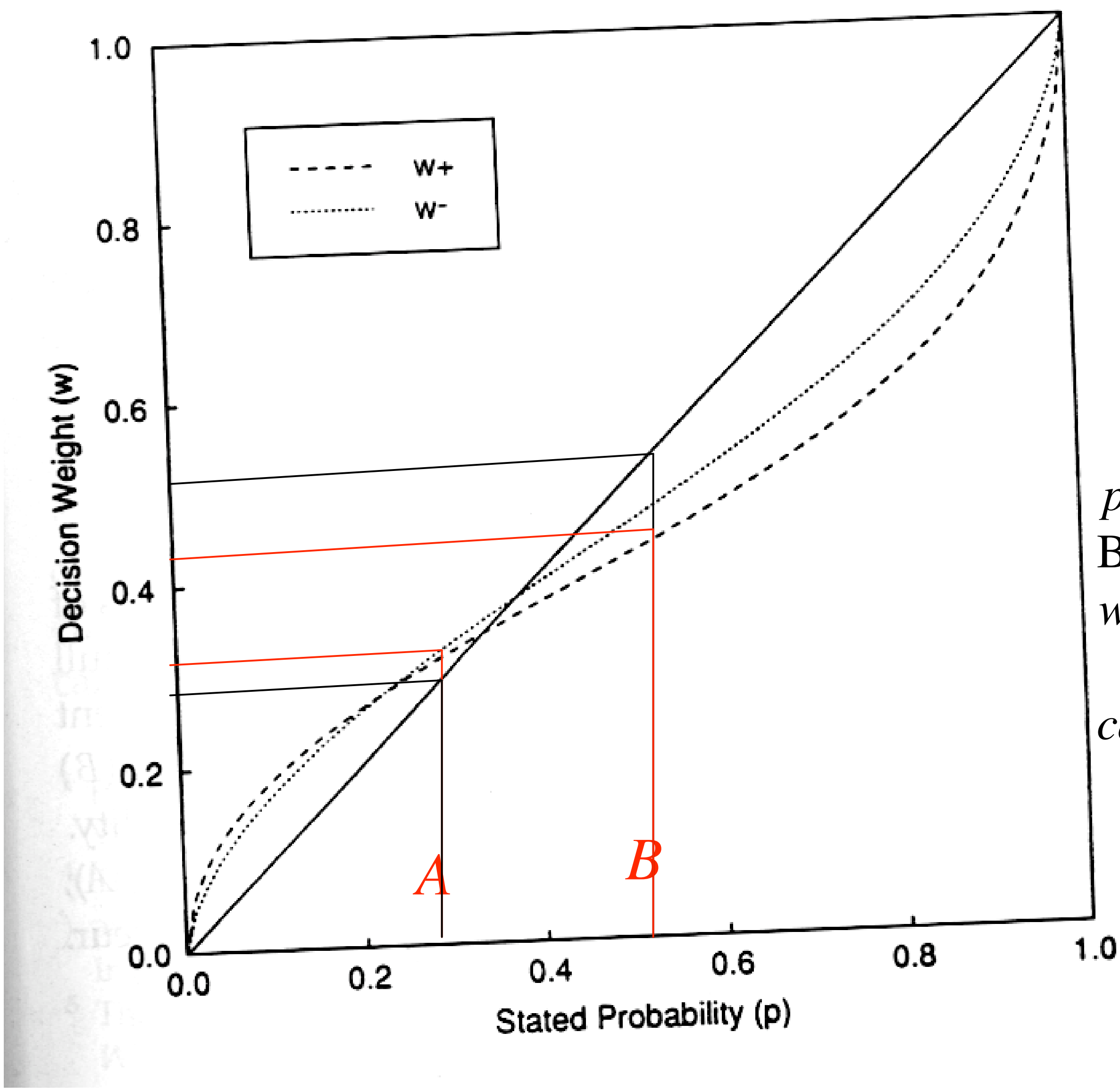


FIGURE 13.3. Flowchart Summarizing the Stages of Decision Making According to Prospect Theory

Distorted Decision Probabilities

Figure 5.1. Weighting functions for gains ($w+$) and losses ($w-$).



Overemphasize small Probabilities

Underemphasize large probabilities

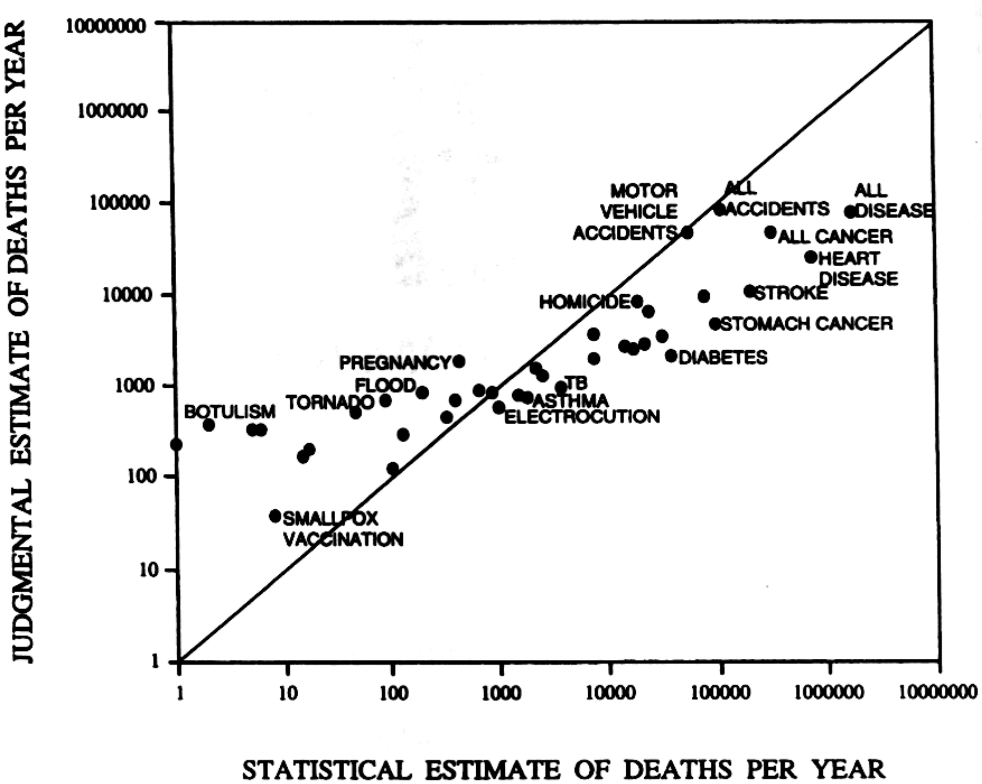
$p(A \cup B) = p(A) + p(B)$ if $A \cap B = \varnothing$

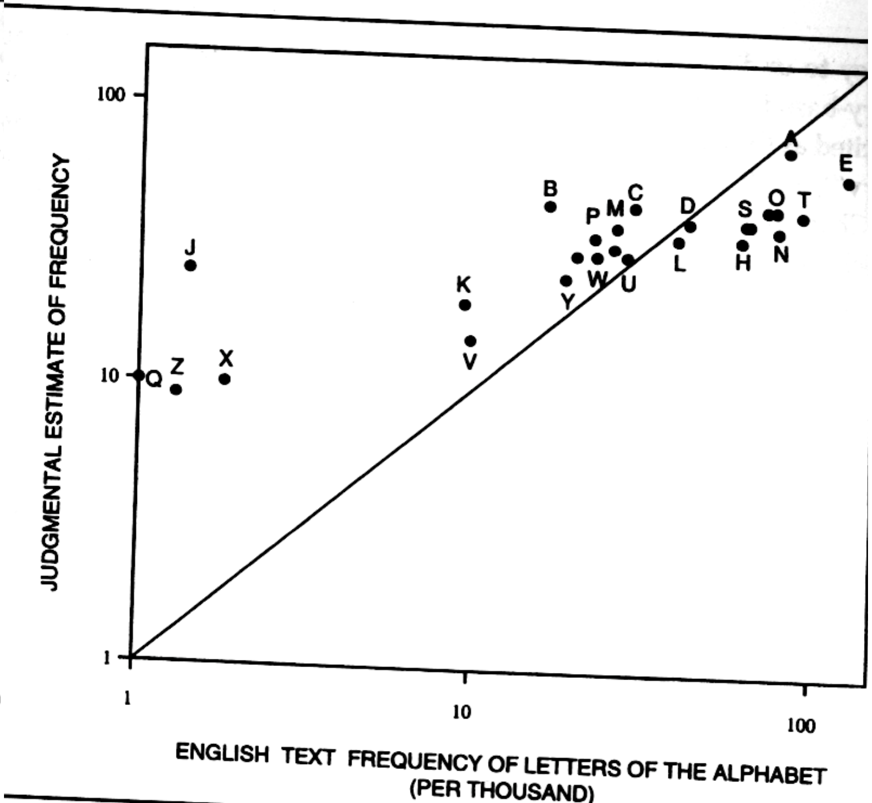
But typically

$w(A \cup B) < w(A) + w(B)$

called Subadditivity

Subjective Probability Estimates





Why biases in Probability assessment?

Uncertainty about beliefs: One view is that people are skeptical--they don't believe the probability numbers given are accurate.

Extreme Cases

Ellsberg Urn Manipulation of Known, Unknown, and Unknowable

Type of Probability	***Cases***
Known Probability	The experimenter filled a bag with 5 red poker chips and 5 black poker chips. You are allowed to examine the bag.
Unknown Probability	The experimenter filled a bag with 10 poker chips that are red and black, but you do not know the relative proportion. You are not allowed to examine the bag.
Unknowable Probability	The experimenter filled a box with 11 bags. The experimenter filled each bag with 10 poker chips that are red and black. Bag 1 has 0 red and 10 black. Bag 2 has 1 red and 9 black and so on. You are allowed to examine the bags. Next, you are asked to draw a bag from the box. The bag you draw is labeled as Bag C. You and the experimenter are not allowed to examine Bag C.

Table 2

A Demonstration of Subadditivity in Betting on the Outcome of a Stanford–Berkeley Football Game

Problem	Option	Events A	Events B	Events C	Events D	Preference (%)
1	f_1	\$25	0	0	0	61
	g_1	0	0	\$10	\$10	39
2	f_2	0	0	0	\$25	66
	g_2	\$10	\$10	0	0	34
3	f_3	\$25	0	0	\$25	29
	g_3	\$10	\$10	\$10	\$10	71

Note. A = Stanford wins by 7 or more points; B = Stanford wins by less than 7 points; C = Berkeley ties or wins by less than 7 points; D = Berkeley wins by 7 or more points. Preference = percentage of respondents (N = 112) that chose each option.

$$f1 > g1 \Rightarrow v(25)W(A) > v(10)W(C \cup D)$$

$$f2 > g2 \Rightarrow v(25)W(D) > v(10)W(A \cup B)$$

$$\Rightarrow$$

$$\frac{W(A)+W(D)}{W(A \cup B)+W(C \cup D)} > \frac{v(10)}{v(25)}$$

$$g3 > f3 \Rightarrow v(10)W(A \cup B \cup C \cup D) > v(25)W(A \cup B)$$

$$\Rightarrow$$

$$\frac{W(A \cup D)}{W(A \cup B \cup C \cup D)} < \frac{v(10)}{v(25)}$$

$$\frac{W(A)+W(D)}{W(A \cup B)+W(C \cup D)} > \frac{W(A \cup D)}{W(A \cup B \cup C \cup D)}$$

Measurement of Decision Weights

Tversky & Fox
40 Football fans
Asked to make
A series of gambles
involving real money:

(25% \$150 or \$40 for sure)

Also had them make a series
Of gambles on Superbowl
games
"Utah wins by up to 12points"

Derived value functions and
extracted Decision weights

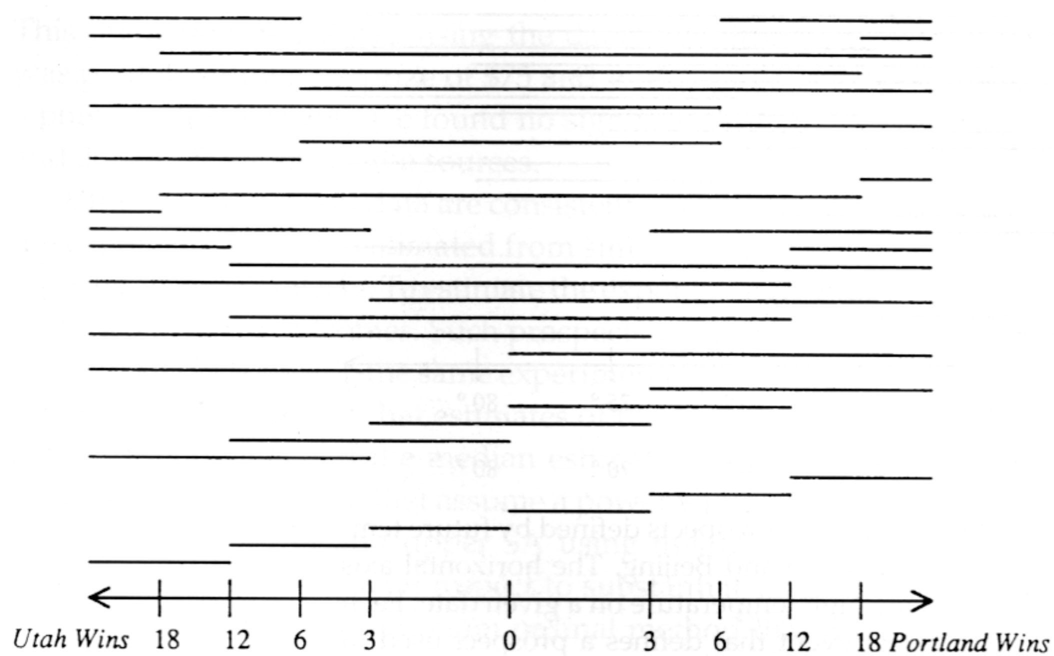


Figure 5.2. Event space for prospects defined by the result of the Utah–Portland basketball game. The horizontal axis refers to the point spread in that game. Each row denotes a target event that defines a prospect used in Study 1. Segments that extend up to the arrowhead represent unbounded intervals. Each interval includes the more extreme endpoint relative to 0, but not the less extreme endpoint.

Procedure. The experiment was run using a computer. Each trial involved a series of choices between a prospect that offered a prize contingent on chance or an uncertain event (e.g., a 25% chance to win a prize of $150) and a descending series of sure payments (e.g., receive $40 for sure). In Study 1, the prize was always $75 for half the respondents and $150 for the other half; in Studies 2 and 3, the prize for all respondents was $150. Certainty equivalents were inferred from two rounds of such choices. The first round consisted of six choices between the prospect and sure payments, spaced roughly evenly between $0 and the prize amount. After completing the first round of choices, a new set of seven sure payments was presented, spanning the narrower range between the lowest payment that the respondent had accepted and highest payment that the respondent had rejected. The program enforced internal consistency. For example, no respondent was allowed to prefer $30 for sure over a prospect and also prefer the same prospect over a sure $40. The program allowed respondents to backtrack if they felt they had made a mistake in the previous round of choices.

The certainty equivalent of each prospect was determined by a linear interpolation between the lowest value accepted and the highest value rejected in the second round of choices. This interpolation yielded a margin of error of ±$2.50 for the $150 prospects and ±$1.25 for the $75 prospects. We wish to emphasize that although our analysis is based

Wakker & Deneffe (1996) Tradeoff Method [Help] [Copyright]

How to use the tradeoff table:

Imagine that your physician tells you that have one of two possible diseases. You have an equal chance of having either Disease 1 or Disease 2. However, your physician is unsure as to which disease you actually have.
Despite the uncertainty as to your condition, your physician thinks it is necessary to proceed with some form of treatment. There are two treatments that can help you. The table below shows the years of life you will obtain under each disease if you choose Treatment 1 or Treatment 2. The green cell represents the number of years of life Treatment 2 will give you if you have Disease 2. This cell is blank and requires of you the following judgment.

Please enter in *green* cell of the table, the number of years of life that Treatment 2 would have to give you if you were to have Disease 2, in order for the two treatments to be of equal preference. Then press <Continue> button. When you press this button, some of the possible outcomes will change. Please repeat the exercise until all values of the years column have been determined..

Start Over					UTILs	Years
	Disease 1		Disease 2		1	
Treatment 1	55		0		2	
Treatment 2	45				3	
					4	
Continue					5	

Trade off Method for Eliciting Standard Utilities

Two gambles:

1) (p, Y, (1-p), r)

   Disease 1, { p=0.5 Y = ? }

   Disease 2, { p=0.5 r = 45 }

$E[U] = p\ U(Y) + (1\text{-}p)\ U(r)$

2) (p, y, (1-p), R)

   Disease 1, { p=0.5 y = 0 }

   Disease 2, { p=0.5 R = 55 }

$E[U] = p\ U(y) + (1\text{-}p)\ U(R)$

Vary Y until subject says the two gambles are equal.

$p\ U(Y) + (1\text{-}p)\ U(r) = p\ U(y) + (1\text{-}p)\ U(R)$

$p\ (U(Y) - U(y)) = (1\text{-}p)\ (U(R) - U(r))$

Perform again with same R & r, but new x.

Again vary X until gambles match

$p\ (U(X) - U(x)) = (1\text{-}p)\ (U(R) - U(r))$

So then

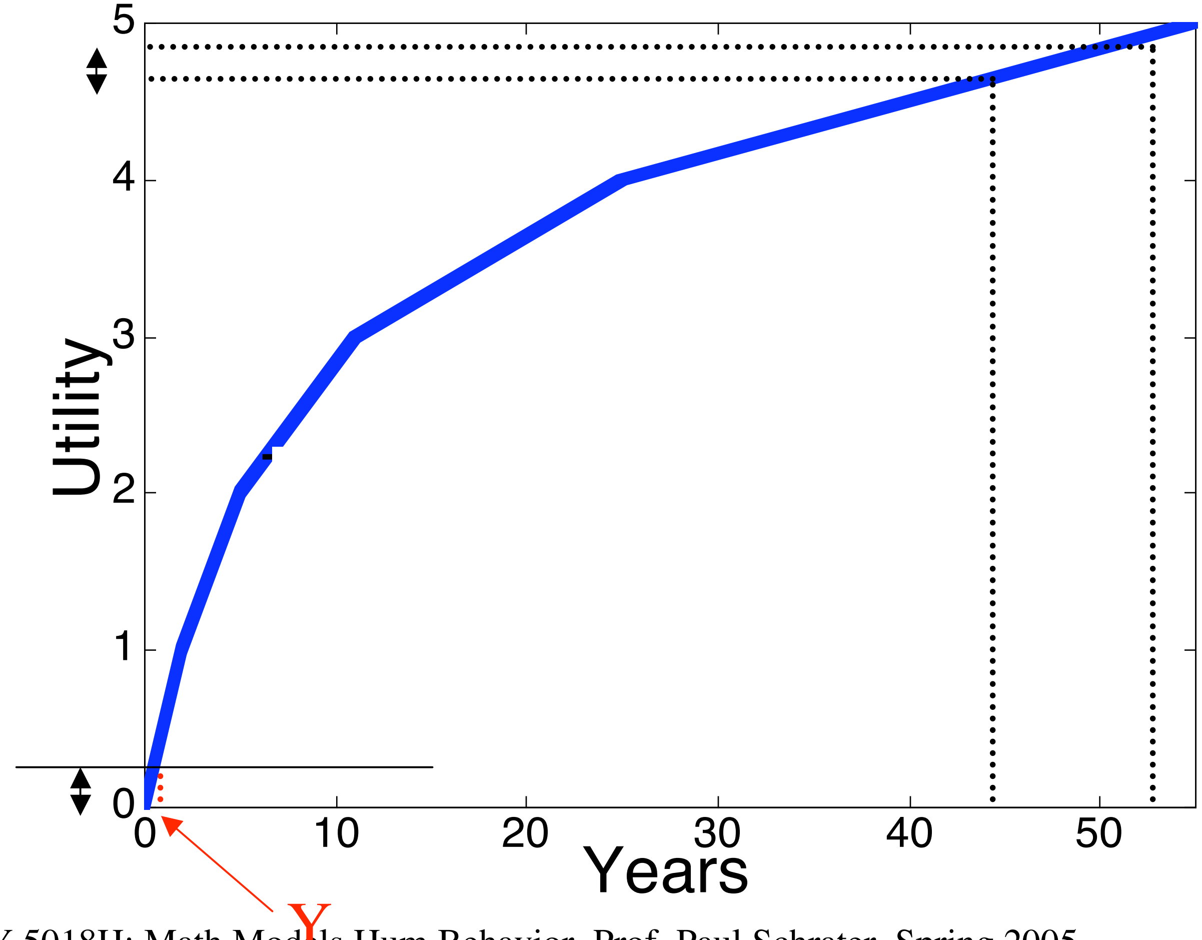
$U(X) - U(x) = U(Y) - U(y)$

Start with y = 0.

Set $U(Y) - U(y) = 1$.

Trade off method

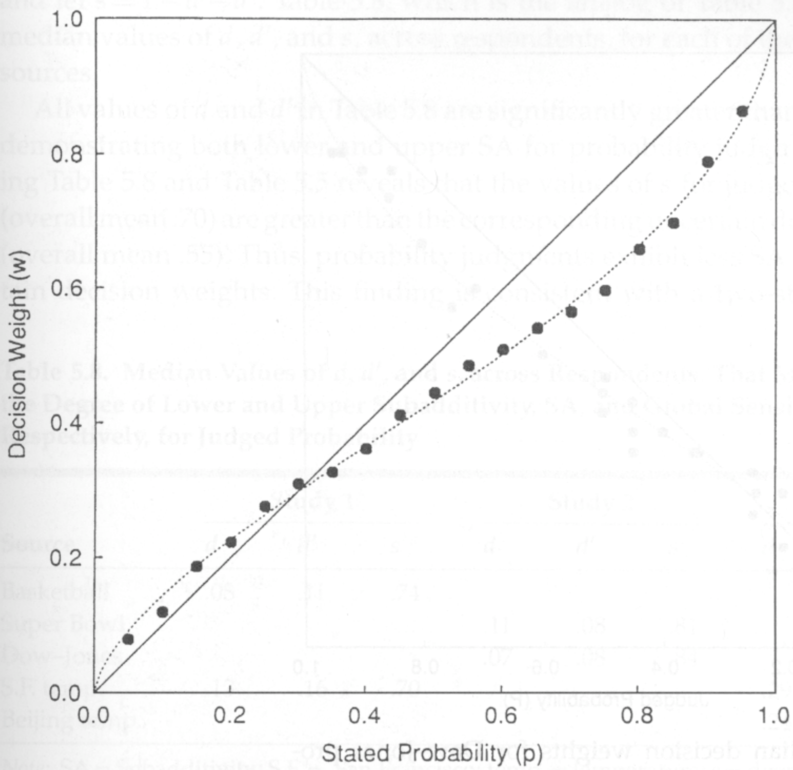
1 unit
Utility



Measured Weights

$$w(\mathrm{p}) = \mathrm{a}\ \mathrm{p}^{d} / (\ \mathrm{a}\ \mathrm{p}^{d} + (1-\mathrm{p})^{d}\ )$$

Figure 5.7. Median decision weights for chance prospects, from Study 2, plotted as a function of stated (objective) probabilities.



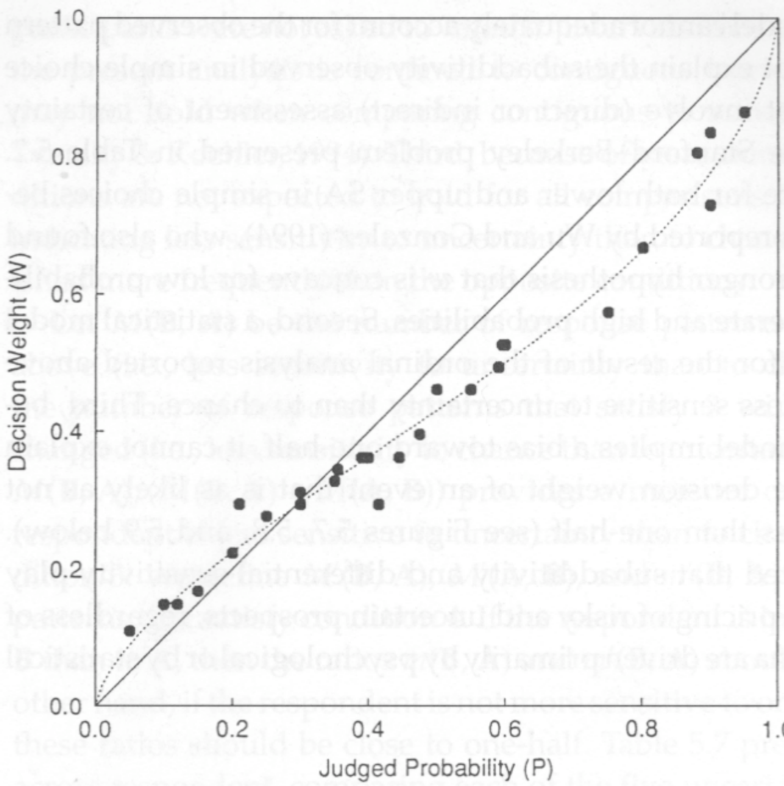
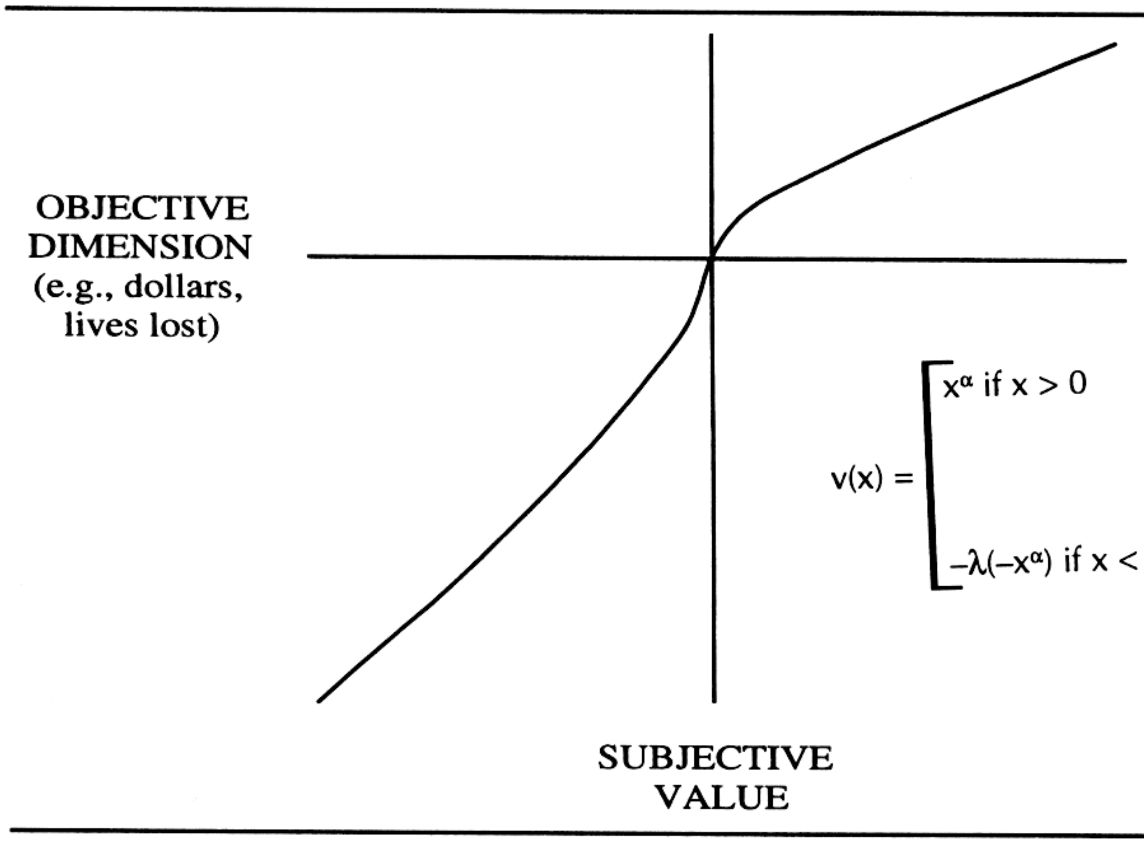


Figure 5.8. Median decision weights for Super Bowl prospects, from Study 2, plotted as a function of median judged probabilities.

Value Function



$$v(x) = \begin{cases} x^{\alpha} \text{ if } x > 0 \\ -\lambda(-x^{\alpha}) \text{ if } x < 0. \end{cases}$$

(with a typical $\alpha = 0.88$ and $\lambda = 2.25$)

FIGURE 13.1. The Prospect Theory Value Function

Gain Loss Framing

PROSPECT A: You receive \$500 for sure.

PROSPECT B: A fair coin is tossed; heads and you receive \$1,000, tails and you receive nothing more.

Now imagine you have just been given \$2,000. Which prospect would you prefer?

PROSPECT A′: You must pay back \$500 immediately.

PROSPECT B′: A fair coin is tossed; heads and you give back nothing, tails and you give back \$1,000.

$$V_{\text{PROSPECT A}} = \pi_{1.00}(v_{+\$500}) = 237.19$$

$$V_{\text{PROSPECT B}} = \pi_{.50}(v_{+\$1000}) + \pi_{.50}(V_0) = 198.18$$

$$V_{\text{PROSPECT A}'} = \pi_{1.00}(v_{-\$500}) = -533.67$$

$$V_{\text{PROSPECT B}'} = \pi_{.50}(V_0) + \pi_{.50}(v_{-\$1000}) = -442.36.$$

Framing a Decision: # Cumulative Prospect Theory

(x, p; y, q)

1) Separate into gains and losses. *For convenience* $|x|>|y|$

2) Compute best case and worst case scenarios

$w^+(p+q)\, v(x) + w^+(q)\, (\, v(y) - v(x)\,)$ $\qquad 0<x<y$

"$p+q$ chance of winning *at least* x and q chance of winning y"

$w^-(p+q)\, v(x) + w^-(q)\, (\, v(y) - v(x)\,)$ $\qquad y<x<0$

"$p+q$ chance of losing *at least* x and q chance of losing y"

$w^-(p)\, v(x) + w^+(q)\, v(y)$ $\qquad x<0<y$

"p chance of losing x and a q chance of gaining y

4-fold Pattern

• Small p, Large gain	Risk seeking
– Lottery playing	
• Small p, Large loss	Risk aversion
– Attractiveness of Insurance	
• Large p, gain	Risk aversion
– Preference for the sure thing	
• Large p, loss	Risk seeking
– Gamble to avoid sure loss	

Table 16.1. Ten Field Phenomena Inconsistent with EU and Consistent with Cumulative Prospect Theory

Domain	Phenomenon	Description	Type of Data	Isolated Decision	Ingredients	References
Stock market	**Equity premium**	Stock returns are too high relative to bond returns	NYSE stock, bond returns	Single yearly return (not long-run)	Loss aversion	Benartzi and Thaler (1995)
Stock market	**Disposition effect**	Hold losing stocks too long, sell winners too early	Individual investor trades	Single stock (not portfolio)	Reflection effect	Odean (in press), Genesove and Mayer (in press)
Labor economics	**Downward-sloping labor supply**	NYC cabdrivers quit around daily income target	Cabdriver hours, earnings	Single day (not week or month)	Loss aversion	Camerer et al. (1997)
Consumer goods	**Asymmetric price elasticities**	Purchases more sensitive to price increases than to cuts	Product purchases (scanner data)	Single product (not shopping cart)	Loss aversion	Hardie, Johnson, Fader(1993)
Macro-economics	**Insensitivity to bad income news**	Consumers do not cut consumption after bad income news	Teachers' earnings, savings	Single year	Loss aversion, reflection effect	Shea (1995); Bowman, Minehart and Rabin (1999)
Consumer choice	**Status quo bias, Default bias**	Consumers do not switch health plans, choose default insurance	Health plan, insurance choices	Single choice	Loss aversion	Samuelson and Zeckhauser (1988), Johnson et al. (1993)
Horse race betting	**Favorite-longshot bias**	Favorites are underbet, longshots overbet	Track odds	Single race (not day)	Overweight low p(loss)	Jullien and Salánie (1997)
Horse race betting	**End-of-the-day effect**	Shift to longshots at the end of the day	Track odds	Single day	Reflection effect	McGlothlin (1956)
Insurance	**Buying phone wire insurance**	Consumers buy overpriced insurance	Phone wire insurance purchases	Single wire risk (not portfolio)	Overweight low p(loss)	Cicchetti and Dubin (1994)
Lottery betting	**Demand for Lotto**	More tickets sold as top prize rises	State lottery sales	Single lottery	Overweight low p(win)	Cook and Clotfelter (1993)