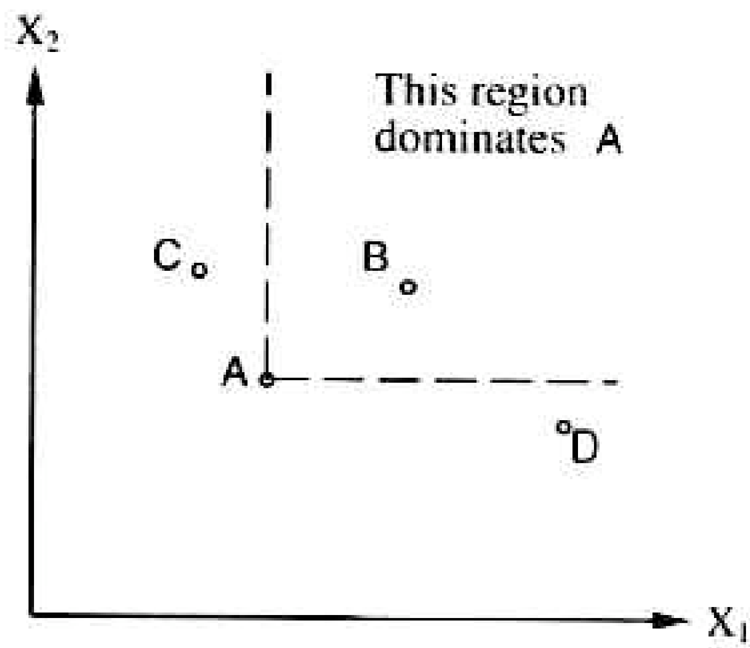
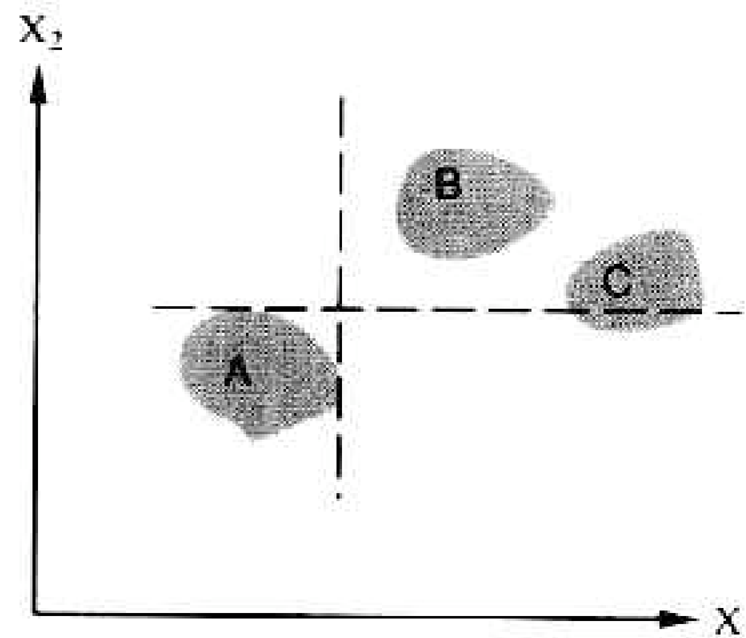
Decision Theory II

Multi-attribute Utility

$$U(x_1, \ldots, x_n) = f[f_1(x_1), \ldots, f_n(x_n)]$$

Hopefully $f_i(x_i)$ are simply like addition





Mapping world properties onto Utility

Some candidate utility measures

Perception:

Correctness of inferences (Signal detection theory)
Perceived energy expenditure (Berkeley's proposal)

Action

success (e.g. Grasp stability (end point accuracy))
Energy expenditure, number of actions

Cognitive

Learning- Generalization error
Problem solutions- Number of possibilities, conditionals

Social

Number of conflicts, number of resources

Perceiving energy expenditure?

Do hills look steeper when you are tired?
Distances farther when laden?

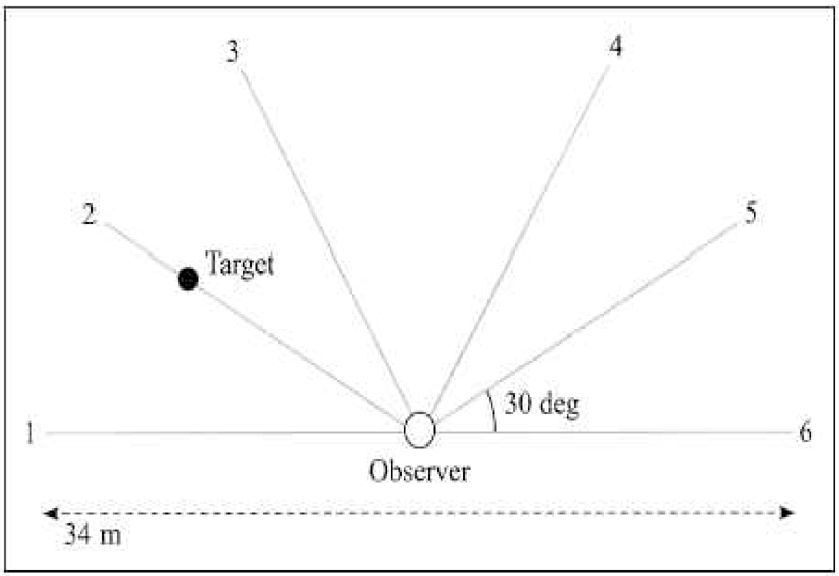


Fig. 1. Bird's-eye view of the target space in Experiment 1. Stimuli were positioned 1 to 17 m from the observer along any of the six radii (1–6).

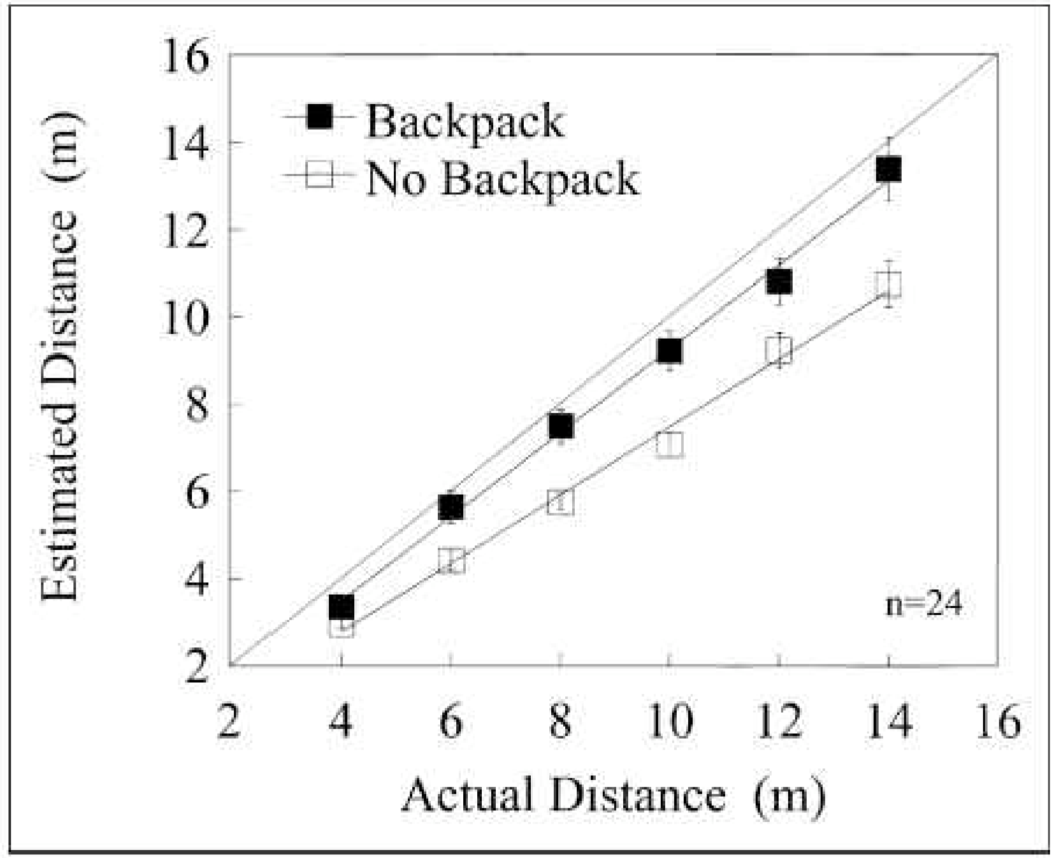


Fig. 2. Estimated distance as a function of actual distance in the backpack and no-backpack conditions of Experiment 1.

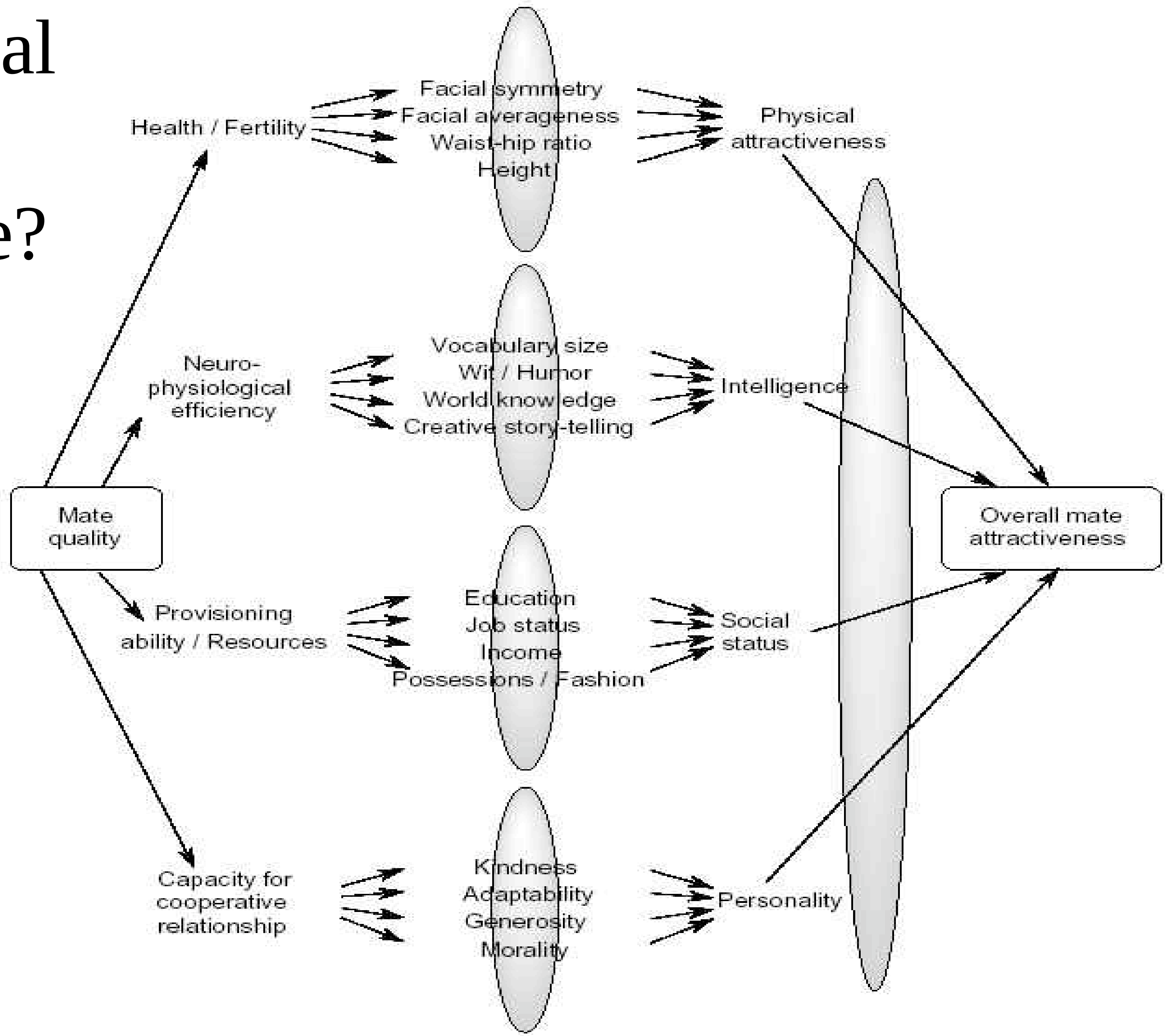
Utility functions for attractiveness?

But what's the use in beauty?

Money?

What else is there?

Rational Mate Choice?



Modeling Belief

- How much do you believe it will rain?
- How strong is your belief in democracy?
- How much do you believe Candidate X?
- How much do you believe Car x is faster than Car y?
- How long do you think you will live?

After your yearly checkup, the doctor has bad news and good news.

The bad news is that you tested positive for a serious disease, and that the test is 99% accurate (i.e. the probability of testing positive given that you have the disease is 0.99, as is the probability of testing negative given that you don't have the disease). The good news is that this is a rare disease, striking only one in 10,000 people.

Why is it good news that the disease is rare?
What are the chances that you actually have the disease?

Posterior Probabilities

- Knowing right prob to compute!
- $P(s|O) = P(O|s)P(s)/P(O)$
  - s = world property
  - O = Observation

- Example: Medical Testing
  - You test positive for cancer
  - A doctor tells you that the test only misses 10% of people who have cancer, so prepare for the worst
  - Do you seek a second opinion? Why?

Mr. Dupont is a professional wine taster. When given a French wine, he will identify it with probability 0.9 correctly as French, and will mistake it for a Californian wine with probability 0.1.

When given a Californian wine, he will identify it with probability 0.8 correctly as Californian, and will mistake it for a French wine with probability 0.2.

Suppose that Mr. Dupont is given ten unlabelled glasses of wine, three with French and seven with Californian wines. He randomly picks a glass, tries the wine, and solemnly says: "French". What is the probability that the wine he tasted was Californian?

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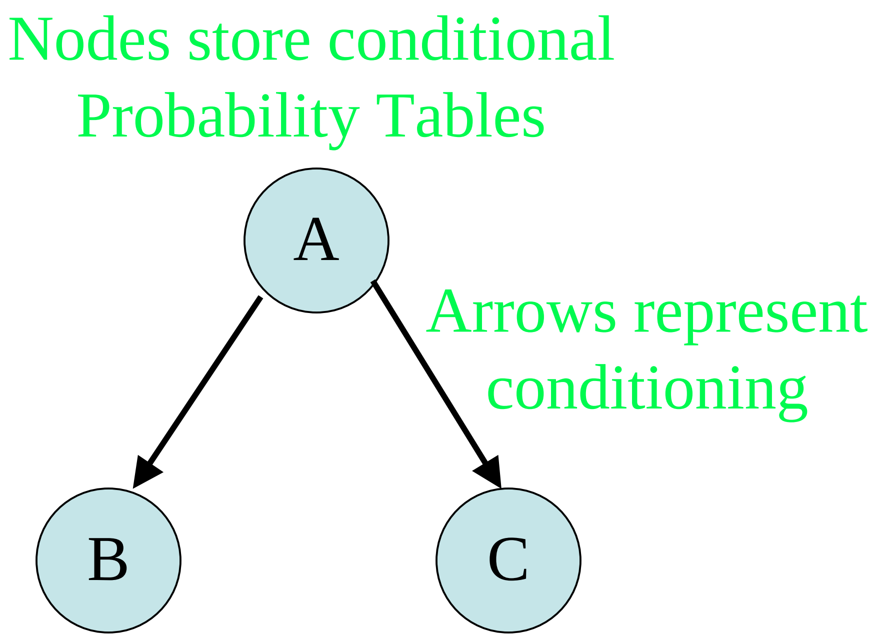
	F	C
Rf	0.9	0.2
Rc	0.1	0.8

$$P(C|Rf) = P(Rf|C)\, p(C)/P(Rf)$$
$$= 0.2*0.7/\sum_w P(Rf|w)p(w)$$
$$= 0.2*0.7/(0.9*0.3+0.2*0.7) = 0.3415$$
$$= 0.2*0.7/0.41 = 0.3415$$

$P(F) = 0.3$; $P(C) = 0.7$;

Graphical Models



This model represents the decomposition:

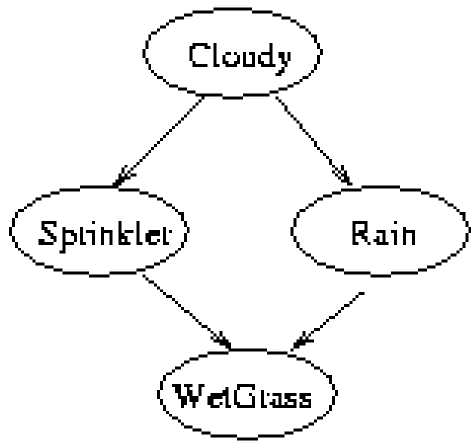
$$P(A,B,C) = P(B|A)\, P(C|A)\, P(A)$$

What would the diagram for $P(B|A,C)\, P(A|C)\, P(C)$ look like?

Sprinkler Problem

You see wet grass.
Did it rain?

P(C=F)	P(C=T)
0.5	0.5



C	P(S=F)	P(S=T)
F	0.5	0.5
T	0.9	0.1

C	P(R=F)	P(R=T)
F	0.8	0.2
T	0.2	0.8

S	R	P(W=F)	P(W=T)
F	F	1.0	0.0
T	F	0.1	0.9
F	T	0.1	0.9
T	T	0.01	0.99

]

Sprinkler Problem cont'd

4 variables: C = Cloudy, R=rain; S=sprinkler; W=wet grass

What's the probability of rain? Need $P(R|W)$

Given: $P(C) = [0.5\ 0.5]$; $P(S|C)$; $P(R|C)$; $P(W|R,S)$

Do the math:

1) ***Get the joint:***

   $P(R,W,S,C) = P(W|R,S)\, P(R|C)\, P(S|C)\, P(C)$

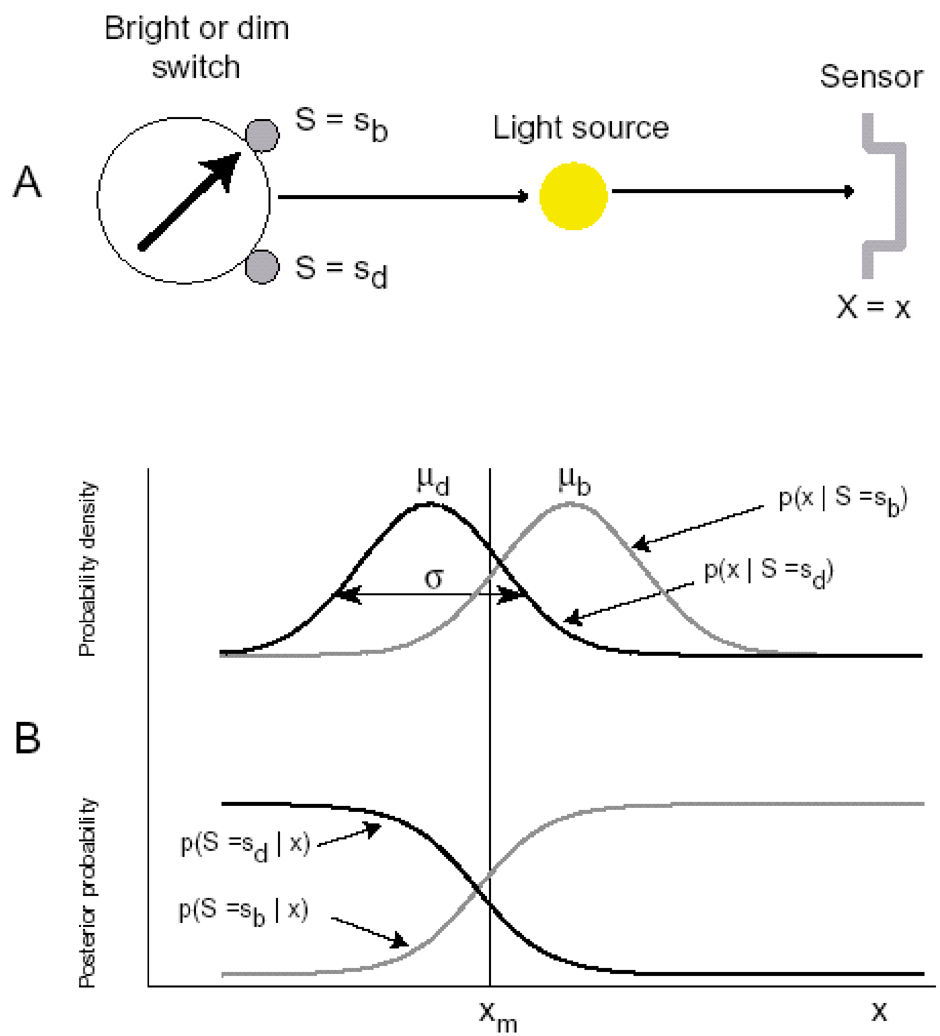
2) ***Marginalize:***

   $P(R,W) = \sum_S \sum_C P(R,W,S,C)$

3) ***Divide:***

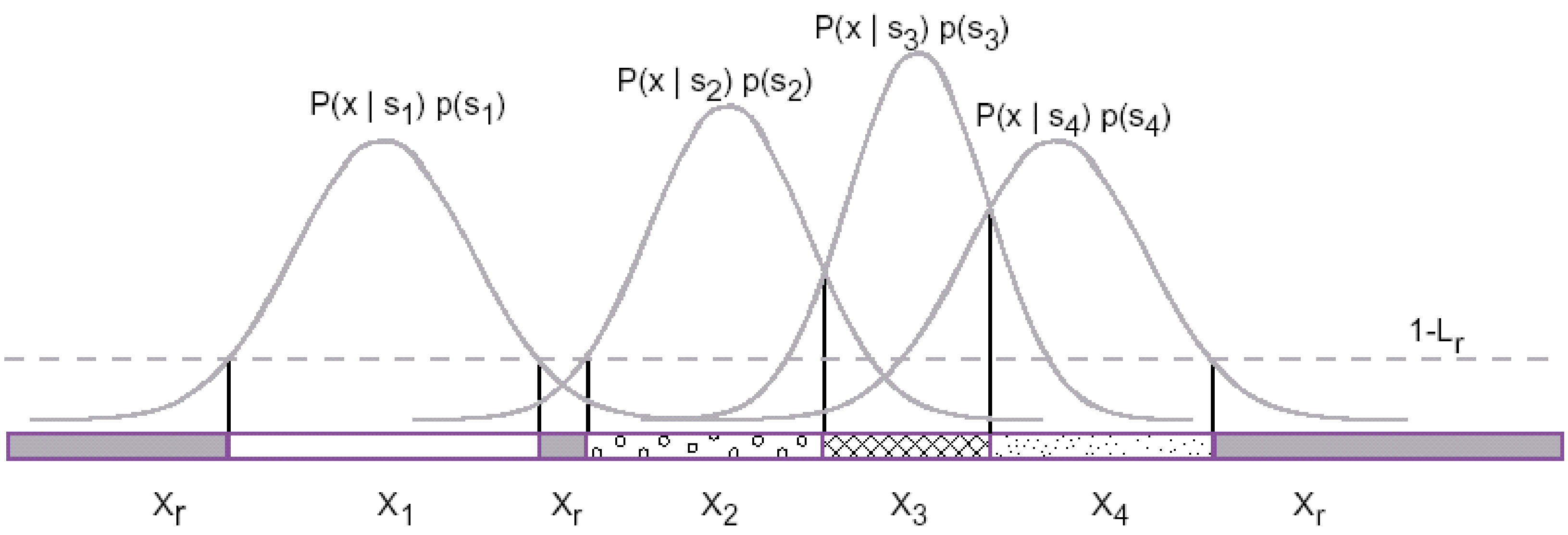
   $P(R|W) = P(R,W)/P(W) = P(R,W)/ \sum_R P(R,W)$

Continuous data, discrete beliefs

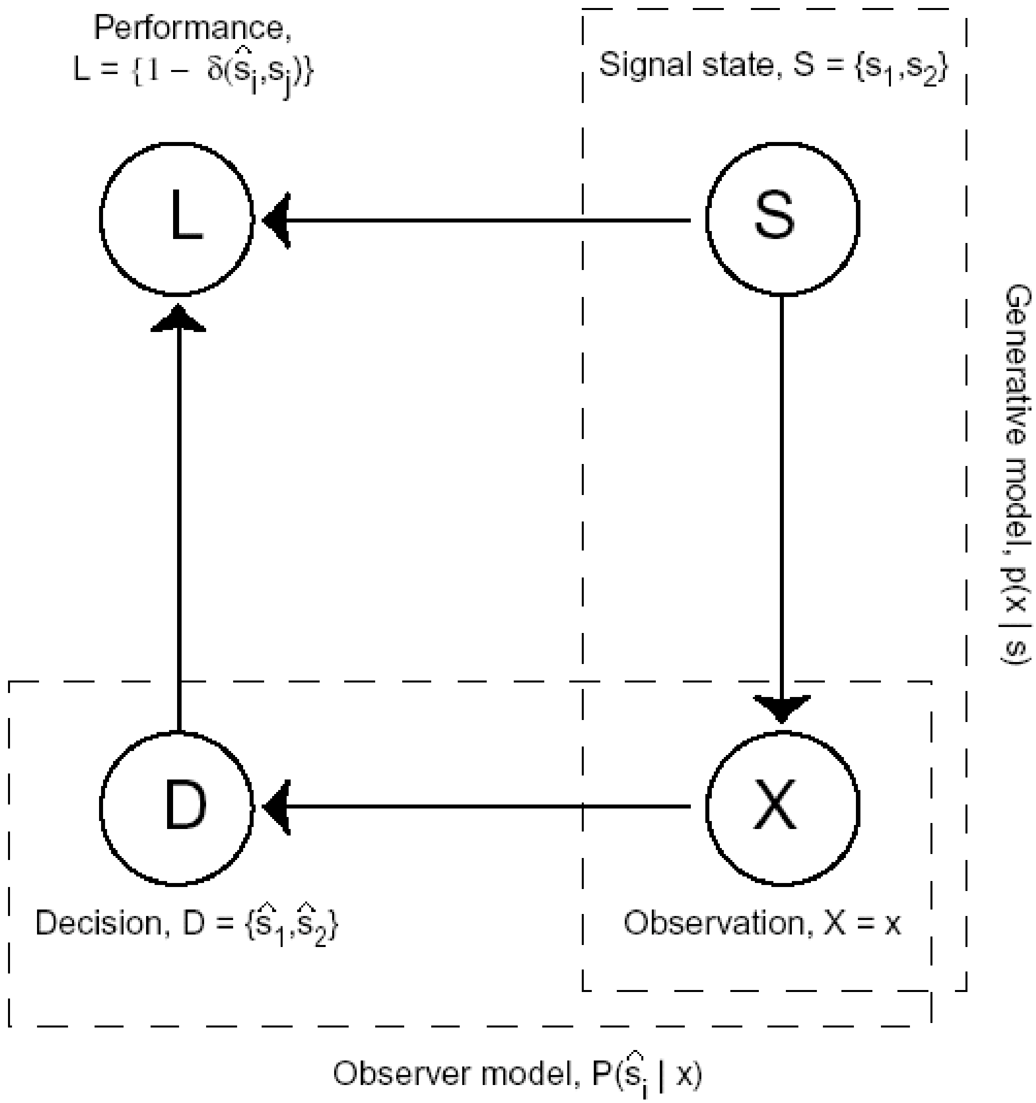


$$P(S = s_b|x) = p(x|S = s_b)(1/3)/p(x), \text{ and } P(S = s_b|x) = p(x|S = s_b)(2/3)/p(x)$$

Continuous data, Multi-class Beliefs



Decision Theory Applied to Human Behavior



Continuous data, continuous beliefs

- Orientation of an object
- Shape
- Attractiveness
- Positions
- Forces
- Time
- Most things