Measurement Theory

Measurement Theory

- Why should we care about Measurement Theory?
  - Correspondence between data and reality
  - Meaningfulness of numbers
  - Provides ways to test assumptions of theory
  - Holds for all scientific data, not just psychological

Measuring Force with a Force Table

$\langle A, \oplus, *, \sim \rangle$

A = Configuration of weight angles

$\oplus$ = physical concatenation

$\sim$ = Comparison procedure

$*$ = Weight scaling

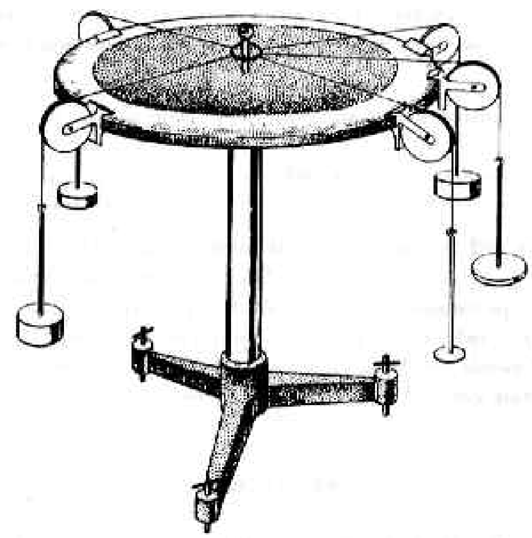
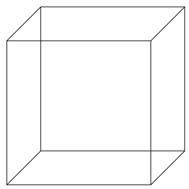


FIG. 1. A force table with an equilibrium configuration.

What does these things measure?

- Personality Tests http://www.outofservice.com/bigfive/
- Mental Rotation times.



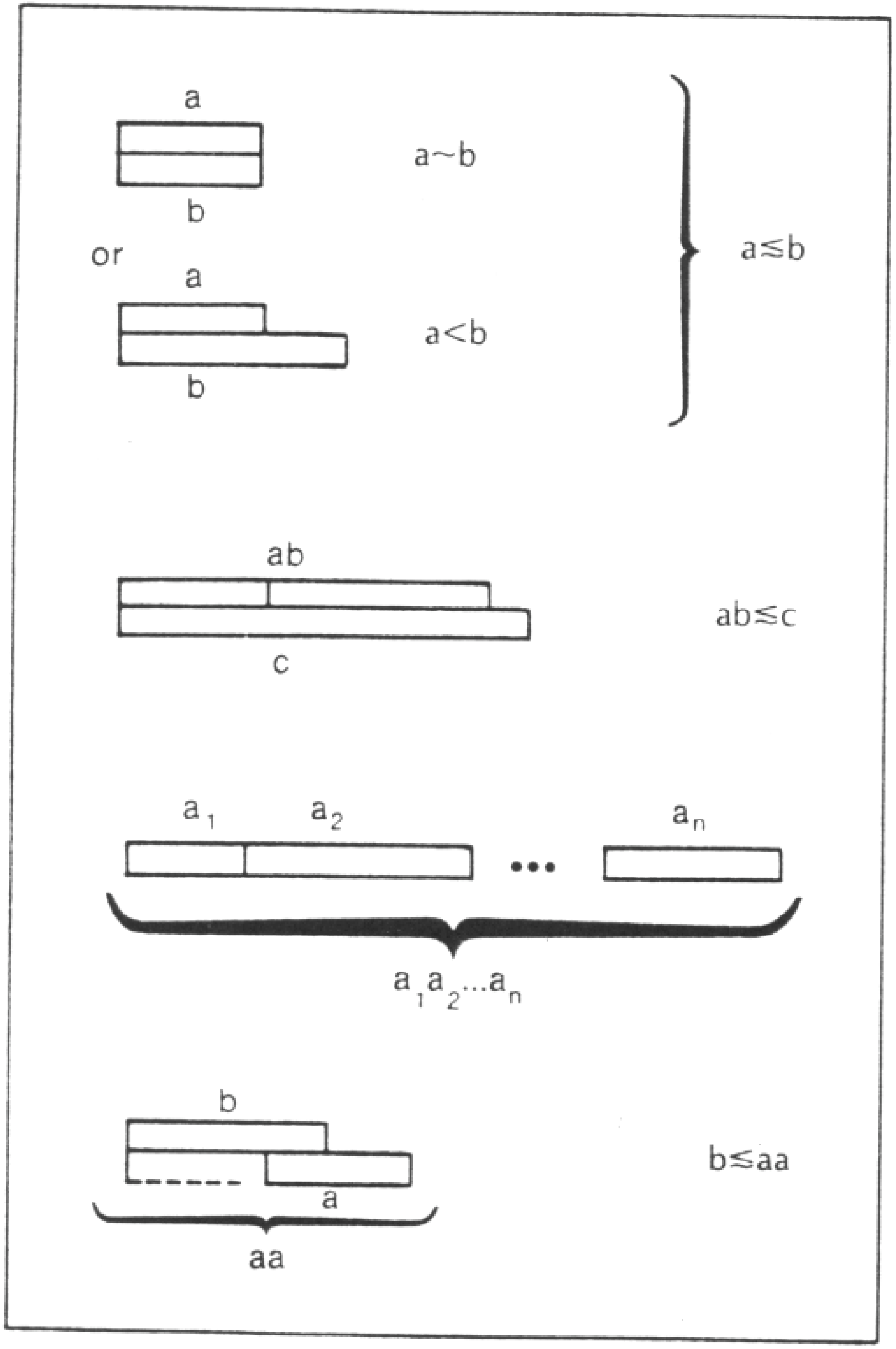
- Reaction times
- Just noticeable differences between the pitch of two tones?
- Stroop test error rates? Blue, Red, Green

Definition

- The representation of properties by numbers/symbols (Coomes, Dawes, Tversky)
  - In addition this definition needs to articulate how world operations map onto numerical operations
- Homomorphism between a qualitative structure and a numerical structure.
  - Homomorphism maps many to one. Isomorphism is one to one. Thus this definition allows for the possibility that there are different operations in the world that will lead to the same measurement.

Measuring Weight

- In the world
  - Objects: $o_1, o_2, \ldots, o_n$ Object labels
  - Relations:
    - Comparison $o_1 (\leq) o_2$ $(\leq)$ denotes comparison by balance
    - Composition o_1 © o_2 © denotes composition by physical juxtaposition
- In numbers
  - Numbers: $w_1, w_2, \ldots, w_n$ Real, denoting weight
  - Relations:
    - $w_1 \leq w_2$ Comparison expressed by logical operation
    - $w_1 + w_2$ Composition expressed by addition



Measuring Length

- In the world
  - Rods: $r_{1,}$ r_2, … , r_n

    Rod labels
  - Relations:

    r_1 (!) r_2 (!) denotes comparison

    r_1 © r_2 © denotes composition by concatenation
- In numbers
  - Numbers: $l_{1,}$ l_2, … , l_n

    Real, denoting length
  - Relations:

    l_1 ! l_2

    $l_1 + l_2$ Composition expressed by addition

Types of Psychological Compositions & Comparisons

- Comparison/ Matching
  - Which of these examples are the same on dimension X, which is largest on dimension X.
  - Bedrock of Psychophysical experiments in perception
- Ranking/Ordering
  - Order this list of movies by your preference. (Bedrock of judgment analysis)
- Production
  - Have the subject produce something via example or instruction. (Copy this picture, Shape your hand to grip a tennis ball)
- Subjective scaling/Magnitude estimation
  - On a scale of 1-X, how bright is that light?

Meaningfulness

- I have an IQ of 200! I must be twice as smart as you. Uhh. I'm worth two of you! I can do math problems twice as fast?! I can join high IQ clubs and contemplate Alien IQs!
- Meaningful Math:
  - numerical operations that can be translated back into valid (true) statements about the world.
  - Valid statements depend on the scale type.
- Transformational approach to measurement scales
    - Absolute => units, origin fixed (e.g. counting)
    - Ratio => units variable, $y = ax$ origin fixed (e.g. mass, length)
    - Interval => change of units $y = a x + b$ (e.g. Fahrenheit to Celsius)
    - Ordinal => Ordering alone
    - Nominal => No ordering, labels only

Building Measurement Scales

- Nominal => "What shall I call thee?"
  - ~ Classification requires an equivalence relation

    A~A Reflexivity

    A~B => B~A Symmetry

    A~B & B~C => A~C Transitivity
- Ordinal => comparison procedure
  - > Requires ability to consistently rank

    A>B => B<A Symmetry

    A>B & B>C => A>C Transitivity
- Interval => Procedure for equating *intervals*

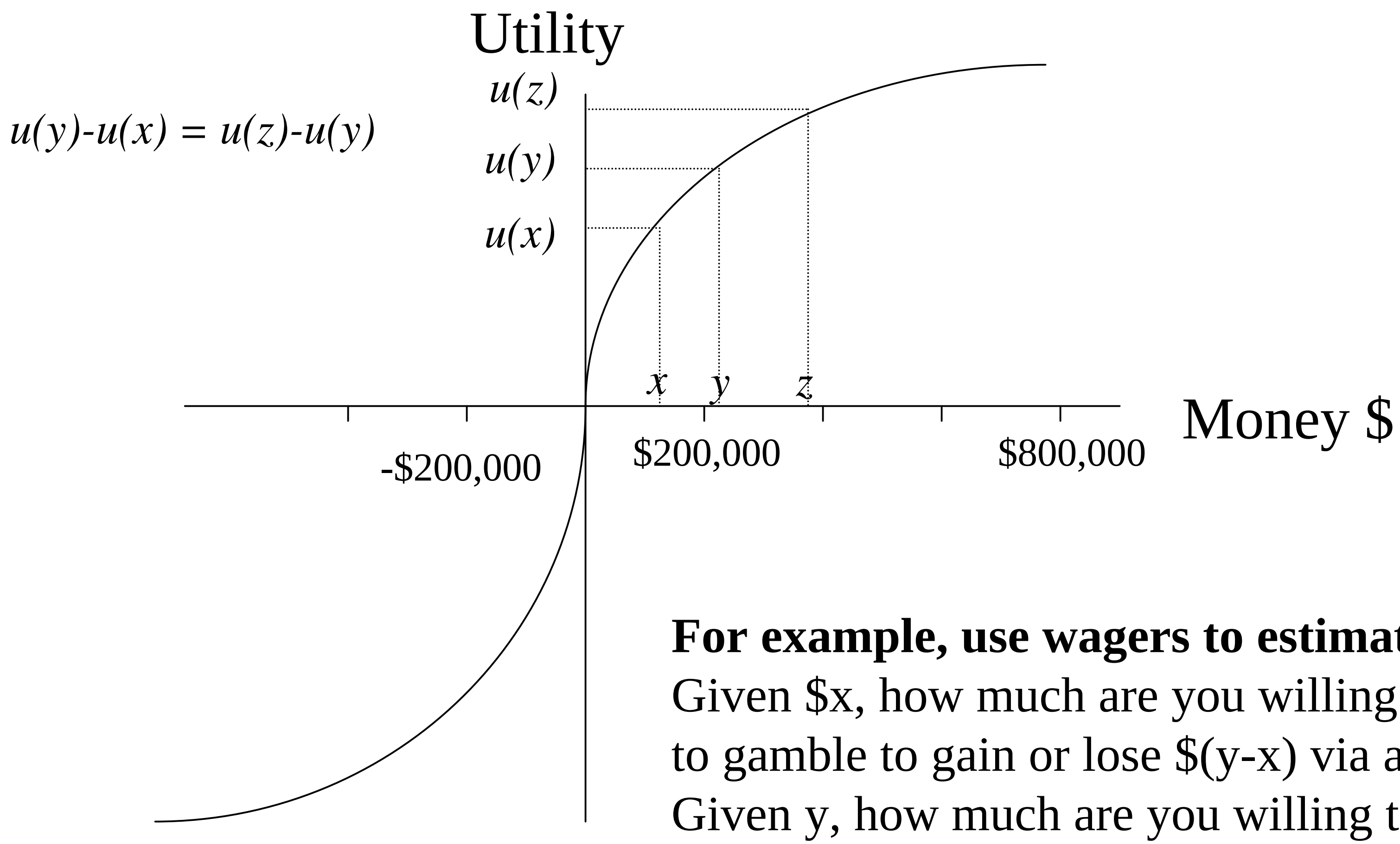
  If u(x) provides the scale, then for three world properties x, y, z

  x > y ~ y > z

  u(x) - u(y) = u(y) - u(z)

  Example: Just noticeable differences

Utility Scale for Money based on gambles for Monetary Gains



For example, use wagers to estimate Utilities:
Given $x, how much are you willing to gamble to gain or lose $(y-x) via a coin-flip.
Given y, how much are you willing to gamble to gain or lose $(z-y) via a coin flip

Fechner's Brightness Scale

- Gustav Theodore Fechner (1802-1887)
  - Shocking! Physicist turns to Psychology! - 1860
- World space and Relations
  - Set of lights of variable intensity L.
  - For any pair of lights with intensities *a,b*
  - Judgment measure: Subject compares brightness of *a* and *b*. *We propose*
    - *a* > *b (a judged greater than b)* if the probability they choose *a* as brighter than *b* > *0.5.*

Fechner's brightness scale cont'd

- Fechner equated intensity differences using equal discrimination probabilities:

  $b > a \sim c > b$ iff $P_{b,a} = P_{c,b}$ *(e.g 0.75)*

  $I_s(b) - I_s(a) = I_s(c) - I_s(b)$ iff $P_{b,a} = P_{c,b}$ *(e.g 0.75)*

  Where $I_s(.)$ is a postulated subjective scale for brightness

- He found:

  $\Delta I = c\, I$ for equal ΔI_s

  If we assume these steps of equal discriminability imply equal increments in subjective brightness ΔI_s, then:

  $\Delta I / \Delta I_s = c\, I$ where $\Delta I_s = 1$ due to method

  Treating $\Delta I / \Delta I_s$ as dI/dI_s $\quad dI/dI_s = c\, I \Rightarrow dI/I = c\; dI_s$

  Integrating:

  $$\int c\, dI_s = \int 1/I\; dI$$

  $$I_s = \tfrac{1}{c}\log(I)$$

Latent Variable Approaches

- Latent variable models:
  - Assume there is a set of causal variables that give rise to the observed behaviors
  - Additionally assume these variables form natural measurement scales
  - Construct models that yield behaviors as functions of the latent variables
  - Regression analysis, factor analysis, most model fitting.
  - Affords natural way of modeling context
- Axiomatic approach:
  - Usually assumes invariant scales can be produced directly in stimulus space.
  - Difficulties modeling context.

What good are IQs?

IQ represents a percentile deviation

- Uses
  - Prediction
    - Even though not an interval scale for intelligence, IQ may play a role as an observed variable that predicts some latent attribute.
  - Description Only
    - Not so useful.

Take away points

- Measurement intrinsically involves theory
  - Latent variable theory
  - Postulation of attributes
- Posited relations in the world *can* be tested
  - Equivalence relations
  - Ordinal relations
  - Interval relations