Color
What is color?

• What do we mean by:
  – Color of an object
  – Color of a light
  – Subjective color impression.
  – Are all these notions the same?

• Wavelengths of light striking the eye are not sufficient or necessary for unique color impression.
• Surface color of an object is usually defined via its reflection function independent of any particular light's spectral distribution.

• Color of a light is usually defined in terms of its spectral distribution.

• Sources of variations in the amount of light at different wavelengths:
  – Light could be produced in different amounts at different wavelengths (compare the sun and a fluorescent light bulb).
  – Light could be differentially reflected (e.g. some pigments).
  – It could be differentially refracted - (e.g. Newton's prism)
  – Wavelength dependent specular reflection - e.g. shiny copper penny (actually most metals).
  – Florescence - light at invisible wavelengths is absorbed and reemitted at visible wavelengths.
Radiometry for colour

• All definitions are now “per unit wavelength”
• All units are now “per unit wavelength”
• All terms are now “spectral”
• Radiance becomes spectral radiance
  – watts per square meter per steradian per unit wavelength
Three problems in color

• Represent surface spectral reflectivity
  – Use set of basis reflectivity functions (pigments)

• Measure light spectra
  – Use pigments with known spectra absorption. Count the amount of light absorbed

• Color inference
  – Find surface reflectivity from measurements of reflected light
Defining Source Color: Black body radiators

• Construct a hot body with near-zero albedo (black body)
  – Easiest way to do this is to build a hollow metal object with a tiny hole in it, and look at the hole.
• The spectral power distribution of light leaving this object is a simple function of temperature

\[ E(\lambda) \propto \left( \frac{1}{\lambda^5} \right) \left( \frac{1}{\exp\left(\frac{hc}{k\lambda T}\right) - 1} \right) \]

• This leads to the notion of color temperature --- the temperature of a black body that would look the same
Measurements of relative spectral power of sunlight, made by J. Parkkinen and P. Silfsten. Relative spectral power is plotted against wavelength in nm. The visible range is about 400nm to 700nm. The color names on the horizontal axis give the color names used for monochromatic light of the corresponding wavelength --- the “colors of the rainbow”.
Relative spectral power of two standard illuminant models --- D65 models sunlight, and illuminant A models incandescent lamps. Relative spectral power is plotted against wavelength in nm. The visible range is about 400nm to 700nm. The color names on the horizontal axis give the color names used for monochromatic light of the corresponding wavelength --- the “colors of the rainbow”.

Violet    Indigo Blue  Green       Yellow       Orange     Red
Measurements of relative spectral power of four different artificial illuminants, made by H. Sugiura. Relative spectral power is plotted against wavelength in nm. The visible range is about 400nm to 700nm.
Spectral albedoes for several different leaves, with color names attached. Notice that different colours typically have different spectral albedo, but that different spectral albedoes may result in the same perceived color (compare the two whites). Spectral albedoes are typically quite smooth functions. Measurements by E.Koivisto.

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The appearance of colors

- Color appearance is strongly affected by (at least):
  - other nearby colors,
  - adaptation to previous views
  - “state of mind”

- We show several demonstrations in what follows.

- Film color mode: View a colored surface through a hole in a sheet, so that the color looks like a film in space; controls for nearby colors, and state of mind.

- Other modes:
  - Surface color
  - Volume color
  - Mirror color
  - Illuminant color
The appearance of colors

- Hering, Helmholtz: Color appearance is strongly affected by other nearby colors, by adaptation to previous views, and by “state of mind”
- Film color mode: View a colored surface through a hole in a sheet, so that the color looks like a film in space; controls for nearby colors, and state of mind.
  - Other modes:
    - Surface colour
    - Volume colour
    - Mirror colour
    - Illuminant colour

- By experience, it is possible to match almost all colors, viewed in film mode using only three primary sources - the principle of trichromacy.
  - Other modes may have more dimensions
    - Glossy-matte
    - Rough-smooth

- Most of what follows discusses film mode.
Radiometry for colour

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The appearance of colors

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- Film color mode:
  View a colored surface through a hole in a sheet, so that the color looks like a film in space; controls for nearby colors, and state of mind.
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    - Volume colour
    - Mirror colour
    - Illuminant colour

- By experience, it is possible to match almost all colors, viewed in film mode using only three primary sources - the principle of **trichromacy**.
  - Other modes may have more dimensions
    - Glossy-matte
    - Rough-smooth

- Most of what follows discusses film mode.
Key idea in color (and vision): Basis functions

Plot shows relative sensitivity as a function of wavelength, for the three cones. The S (for short) cone responds most strongly at short wavelengths; the M (for medium) at medium wavelengths and the L (for long) at long wavelengths.

These are occasionally called B, G and R cones respectively, but that’s misleading - you don’t see red because your R cone is activated.
What are basis functions?

• We need flexible method for constructing a function $f(t)$ out of a predefined set of other functions.
• We pick a system of $K$ basis functions $g_k(t)$, and call this the basis for $f(t)$.
• We express $f(t)$ as a weighted sum of these basis functions:

$$f(t) = a_1 g_1(t) + a_2 g_2(t) + \ldots + a_K g_K(t)$$

The coefficients $a_1, \ldots, a_K$ determine the shape of the function.
Representing functions as sums of other functions: Synthesis

\[ f(x) = \sum_{i=1:N} a_i g_i(x) \]

Sampled function

Basis functions

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Representing functions as sums of other functions: Analysis

Given what are the $a_i$?

Project function onto Basis function

$$a_i = \int_{-\infty}^{\infty} f(x) g_i(x) \, dx$$
Photopigments are basis functions for spectral distributions.

Plot shows relative sensitivity as a function of wavelength, for the three cones. The S (for short) cone responds most strongly at short wavelengths; the M (for medium) at medium wavelengths and the L (for long) at long wavelengths.

These are occasionally called B, G and R cones respectively, but that’s misleading - you don’t see red because your R cone is activated.
Human color representation is a linear basis function system

Color matching experiments - I

- Show a split field to subjects; one side shows the light whose color one wants to measure, the other a weighted mixture of primaries (fixed lights).
- Each light is seen in film color mode.
Color matching experiments - II

• Many colors can be represented as a mixture of A, B, C lights with different spectral distributions
• write
  \[ M = aA + bB + cC \]
  where the = sign should be read as “matches”
• This is additive matching.
• Gives a color description system - two people who agree on A, B, C need only supply \((a, b, c)\) to describe a color.
Subtractive matching

• Some colors can’t be matched like this: instead, must write

\[ M + a \ A = b \ B + c \ C \]

• This is **subtractive** matching.

• Interpret this as \((-a, b, c)\)

• Problem for building monitors: Choose R, G, B such that positive linear combinations match a large set of colors
The principle of trichromacy

- Experimental facts:
  - Three primaries will work for most people if we allow subtractive matching
    - Exceptional people can match with two or only one primary.
    - This could be caused by a variety of deficiencies.
  - Most people make the same matches.
    - There are some anomalous trichromats, who use three primaries but make different combinations to match
    - These matches can be used to determine the genetics of their photopigments.
Grassman’s Laws

• For colour matches made in film colour mode:
  – symmetry: \( U=V \iff V=U \)
  – transitivity: \( U=V \) and \( V=W \) \( \implies \) \( U=W \)
  – proportionality: \( U=V \iff tU=tV \)
  – additivity: if any two (or more) of the statements
    \( U=V, \)
    \( W=X, \)
    \( (U+W)=(V+X) \) are true, then so is the third

• These statements are as true as any biological law. They mean that color matching in film color mode is linear.
Linear color spaces

• A choice of primaries yields a linear color space --- the coordinates of a color are given by the weights of the primaries used to match it.

• Choice of primaries is equivalent to choice of color space.

• **RGB:** primaries are monochromatic energies are 645.2nm, 526.3nm, 444.4nm.

• **CIE XYZ:** Primaries are imaginary, but have other convenient properties. Color coordinates are \((X,Y,Z)\), where \(X\) is the amount of the \(X\) primary, etc.
  
  – Usually draw \(x, y\), where
    
    \[
    x = \frac{X}{X+Y+Z} \\
    y = \frac{Y}{X+Y+Z}
    \]
Color matching functions

- Choose primaries, say A, B, C
- Given energy function, what amounts of primaries will match it?
- For each wavelength, determine how much of A, of B, and of C is needed to match light of that wavelength alone.

\[
E(\lambda) \quad \begin{array}{c}
\text{a(}\lambda\text{)} \\
\text{b(}\lambda\text{)} \\
\text{c(}\lambda\text{)}
\end{array}
\]

Then our match is:

\[
\left\{ \int a(\lambda) E(\lambda) d\lambda \right\} A + \\
\left\{ \int b(\lambda) E(\lambda) d\lambda \right\} B + \\
\left\{ \int c(\lambda) E(\lambda) d\lambda \right\} C
\]
RGB: primaries are monochromatic, energies are 645.2nm, 526.3nm, 444.4nm. Color matching functions have negative parts -> some colors can be matched only subtractively.
CIE XYZ: Color matching functions are positive everywhere, but primaries are not physical lights. Usually draw x, y, where
\[ x = \frac{X}{X+Y+Z} \]
\[ y = \frac{Y}{X+Y+Z} \]
Color receptors

- **Principle of univariance**: cones give the same kind of response, in different *amounts*, to different wavelengths. The output of the cone is obtained by summing over wavelengths. Responses are measured in a variety of ways (comparing behaviour of color normal and color deficient subjects).

- All experimental evidence suggests that the response of the k’th type of cone can be written as

\[ \int \rho_k(\lambda) E(\lambda) d\lambda \]

where \( \rho_k(\lambda) \) is the sensitivity of the receptor and spectral energy density of the incoming light.
Color receptors

Plot shows relative sensitivity as a function of wavelength, for the three cones. The S (for short) cone responds most strongly at short wavelengths; the M (for medium) at medium wavelengths and the L (for long) at long wavelengths.

These are occasionally called B, G and R cones respectively, but that’s misleading - you don’t see red because your R cone is activated.
Simpler if we discretize frequency

\[ E(\lambda) \]
What does color matching do?

Choose

\[
\begin{bmatrix}
  f_1 \\
  f_2 \\
  f_3
\end{bmatrix} =
\begin{bmatrix}
  \vec{a}^T \\
  \vec{b}^T \\
  \vec{c}^T
\end{bmatrix} \tilde{E}
\]

Such that when:

\[
\begin{bmatrix}
  I_r \\
  I_g \\
  I_b
\end{bmatrix} = \begin{bmatrix}
  \tilde{\rho}_r^T \\
  \tilde{\rho}_g^T \\
  \tilde{\rho}_b^T
\end{bmatrix} \tilde{E}
\]

Then the combined primaries yields:

\[
\begin{bmatrix}
  I_r \\
  I_g \\
  I_b
\end{bmatrix} = \begin{bmatrix}
  \tilde{\rho}_r^T \\
  \tilde{\rho}_g^T \\
  \tilde{\rho}_b^T
\end{bmatrix} \begin{bmatrix}
  \vec{A} & \vec{B} & \vec{C}
\end{bmatrix} \begin{bmatrix}
  f_1 \\
  f_2 \\
  f_3
\end{bmatrix}
\]
Color matching functions

Choose

\[
\begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3
\end{bmatrix}
= \begin{bmatrix}
    \vec{a}^T \\
    \vec{b}^T \\
    \vec{c}^T
\end{bmatrix} \tilde{E}
\]

Then the combined primaries yields:

\[
\begin{bmatrix}
    \vec{\rho}_r^T \\
    \vec{\rho}_g^T \\
    \vec{\rho}_b^T
\end{bmatrix} \tilde{E} = \begin{bmatrix}
    \vec{\rho}_r^T \\
    \vec{\rho}_g^T \\
    \vec{\rho}_b^T
\end{bmatrix} \begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3
\end{bmatrix} \Rightarrow \begin{bmatrix}
    \vec{\rho}_r^T \\
    \vec{\rho}_g^T \\
    \vec{\rho}_b^T
\end{bmatrix} \tilde{E} = M \begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3
\end{bmatrix}
\]

\[
M^{-1} \begin{bmatrix}
    \vec{\rho}_r^T \\
    \vec{\rho}_g^T \\
    \vec{\rho}_b^T
\end{bmatrix} \tilde{E} = \begin{bmatrix}
    f_1 \\
    f_2 \\
    f_3
\end{bmatrix} \Rightarrow M^{-1} \begin{bmatrix}
    \vec{\rho}_r^T \\
    \vec{\rho}_g^T \\
    \vec{\rho}_b^T
\end{bmatrix} = \begin{bmatrix}
    \vec{a}^T \\
    \vec{b}^T \\
    \vec{c}^T
\end{bmatrix}
\]
A qualitative rendering of the CIE (x,y) space. The blobby region represents visible colors. There are sets of (x, y) coordinates that don’t represent real colors, because the primaries are not real lights (so that the color matching functions could be positive everywhere).
A plot of the CIE (x,y) space. We show the spectral locus (the colors of monochromatic lights) and the black-body locus (the colors of heated black-bodies). I have also plotted the range of typical incandescent lighting.
Non-linear colour spaces

- HSV: Hue, Saturation, Value are non-linear functions of XYZ.  
  – because hue relations are naturally expressed in a circle

- Uniform: equal (small!) steps give the same perceived color changes.

- Munsell: describes surfaces, rather than lights. Surfaces must be viewed under fixed comparison light
HSV hexcone

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Uniform color spaces

• McAdam ellipses (next slide) demonstrate that differences in x,y are a poor guide to differences in color

• Construct color spaces so that differences in coordinates are a good guide to differences in color.
Variations in color matches on a CIE x, y space. At the center of the ellipse is the color of a test light; the size of the ellipse represents the scatter of lights that the human observers tested would match to the test color; the boundary shows where the just noticeable difference is. The ellipses on the left have been magnified 10x for clarity; on the right they are plotted to scale. The ellipses are known as MacAdam ellipses after their inventor. The ellipses at the top are larger than those at the bottom of the figure, and that they rotate as they move up. This means that the magnitude of the difference in x, y coordinates is a poor guide to the difference in color.
CIE $u'v'$ which is a projective transform of $x, y$. We transform $x,y$ so that ellipses are most like one another. Figure shows the transformed ellipses.
Color constancy

• Assume we’ve identified and removed specularities
• The spectral radiance at the camera depends on two things
  – surface albedo
  – illuminant spectral radiance
  – the effect is much more pronounced than most people think (see following slides)
• We would like an illuminant invariant description of the surface
  – e.g. some measurements of surface albedo
  – need a model of the interactions
Notice how the color of light at the camera varies with the illuminant color; here we have a uniform reflectance illuminated by five different lights, and the result plotted on CIE x, y.
Notice how the color of light at the camera varies with the illuminant color; here we have the blue flower illuminated by five different lights, and the result plotted on CIE x,y. Notice how it looks significantly more saturated under some lights.
Notice how the color of light at the camera varies with the illuminant color; here we have a green leaf illuminated by five different lights, and the result plotted on CIE x,y.
Viewing coloured objects

- Assume diffuse + specular model

- Specular
  - specularities on dielectric objects take the colour of the light
  - specularities on metals can be coloured

- Diffuse
  - colour of reflected light depends on both illuminant and surface
  - people are surprisingly good at disentangling these effects in practice (colour constancy)
  - this is probably where some of the spatial phenomena in colour perception come from
When one views a colored surface, the spectral radiance of the light reaching the eye depends on both the spectral radiance of the illuminant, and on the spectral albedo of the surface. We’re assuming that camera receptors are linear, like the receptors in the eye. This is usually the case.
Diffuse region

Boundary of specularity
Figure 1. Same objects imaged under two natural illuminants. Top) The patches show a rectangular region extracted from images of the same object under different outdoor illuminants. Bottom) The images from which the patches were taken. Images were acquired by the author in Merion Station, PA using a Nikon CoolPix 995 digital camera. The automatic white balancing calculation that is a normal part of the camera’s operation was disabled during image acquisition.
Gradient: The two patches shown were extracted from the upper left (L) and lower right (R; above table) of the back wall of the scene. Shadow: The two patches were extracted from the tabletop in direct illumination (D) and shadow (S). Shape: The three patches shown were extracted from two regions of the sphere (T and B; center top and right bottom respectively) and from the colored panel directly above the sphere (P; the panel is the leftmost of the four in the bottom row). Both the sphere and panel have the same simulated surface reflectance function. Pose and Indirect Illum. The four patches were extracted from the three visible sides of the cube (R, L, and T; right, left and top visible sides respectively) and from the left side of the folded paper located between the cube and sphere (I). The simulated surface reflectance of all sides of the cube and of the left side of the folded paper are identical. The image was
Land’s Demonstration

White light

Photometer reading
(1, 0.3, 0.3)

Audience name
"Red"

Coloured light

Photometer reading
(1, 0.3, 0.3)

Audience name
"Blue"
Lightness Constancy

• Lightness constancy
  – how light is the surface, independent of the brightness of the illuminant
  – issues
    • spatial variation in illumination
    • absolute standard
  – Human lightness constancy is very good

• Assume
  – frontal 1D “Surface”
  – slowly varying illumination
  – quickly varying surface reflectance
Thresholded $\frac{d \log p}{dx}$

Integrate
This to get
Lightness Constancy in 2D

- Differentiation, thresholding are easy
  - integration isn’t
  - problem - gradient field may no longer be a gradient field
- One solution
  - Choose the function whose gradient is “most like” thresholded gradient

- This yields a minimization problem
- How do we choose the constant of integration?
  - average lightness is grey
  - lightest object is white
  - ?
Simplest colour constancy

• Adjust three receptor channels independently
  – Von Kries
  – Where does the constant come from?
    • White patch
    • Averages
    • Some other known reference (faces, nose)
Colour Constancy - I

- We need a model of interaction between illumination and surface colour
  - finite dimensional linear model seems OK

- Finite Dimensional Linear Model (or FDLM)
  - surface spectral albedo is a weighted sum of basis functions
  - illuminant spectral exitance is a weighted sum of basis functions
  - This gives a quite simple form to interaction between the two
Finite Dimensional Linear Models

\[ E(\lambda) = \sum_{i=1}^{m} \varepsilon_i \psi_i(\lambda) \]

Incoming spectral radiance \( E(\lambda) \)

Receptor response of \( k \)'th receptor class

\[ \int_{\lambda} \sigma(\lambda) \rho(\lambda) E(\lambda) d\lambda \]

Outgoing spectral radiance \( E(\lambda) \rho(\lambda) \)

Spectral albedo \( \rho(\lambda) \)

\[ \rho(\lambda) = \sum_{j=1}^{n} r_j \varphi_j(\lambda) \]

\[ p_k = \int_{\lambda} \sigma_k(\lambda) \left( \sum_{i=1}^{m} \varepsilon_i \psi_i(\lambda) \right) \left( \sum_{j=1}^{n} r_j \varphi_j(\lambda) \right) d\lambda \]

= \sum_{i=1, j=1}^{m,n} \varepsilon_i r_j \int_{\lambda} \sigma_k(\lambda) \psi_i(\lambda) \varphi_j(\lambda) d\lambda

= \sum_{i=1, j=1}^{m,n} \varepsilon_i r_j g_{ijk} \]
General strategies

• Determine what image would look like under white light

• Assume
  – that we are dealing with flat frontal surfaces
  – We’ve identified and removed specularities
  – no variation in illumination

• We need some form of reference
  – brightest patch is white
  – spatial average is known
  – gamut is known
  – specularities
Obtaining the illuminant from specularities

• Assume that a specularity has been identified, and material is dielectric.

• Then in the specularity, we have

\[ p_k = \int \sigma_k(\lambda)E(\lambda)d\lambda \]

\[ = \sum_{i=1}^{m} \varepsilon_i \int \sigma_k(\lambda)\psi_i(\lambda)d\lambda \]

(unfortunately, not true)
Obtaining the illuminant from average color assumptions

- Assume the spatial average reflectance is known

- We can measure the spatial average of the receptor response to get

\[
\bar{\rho}(\lambda) = \sum_{j=1}^{n} r_j \varphi_j(\lambda)
\]

- Assuming
  - \(g_{ijk}\) are known
  - average reflectance is known
  - there are not more receptor types than illuminant basis functions

- We can recover the illuminant coefficients from this linear system

\[
\bar{p}_k = \sum_{i=1,j=1}^{m,n} \varepsilon_i r_j g_{ijk}
\]
Computing surface properties

- Two strategies
  - compute reflectance coefficients
  - compute appearance under white light.
- These are essentially equivalent.
- Once illuminant coefficients are known, to get reflectance coefficients we solve the linear system

\[
p_k = \sum_{i=1,j=1}^{m,n} \varepsilon_i r_j g_{ijk}
\]

- to get appearance under white light, plug in reflectance coefficients and compute

\[
p_k = \sum_{i=1,j=1}^{m,n} \varepsilon_i \varepsilon_i^{\text{white}} r_j g_{ijk}
\]
Bayesian Color Constancy

\[
p(x|y) = \frac{p(y|x)p(x)}{p(y)} = Cp(y|x)p(x).
\]

\[
\bar{L}(\tilde{x}|y) = \int_{x} L(\tilde{x}, x)p(x|y)dx.
\]

\[
s_k = \sum_{i=1, j=1}^{m, n} \epsilon_i r_j g_{ijk}
\]

\[
y_k = f(x), \quad x = \epsilon_i r_j
\]

Simplest example:

\[
y = ab
\]

\[
p(y|x) = \frac{1}{\sqrt{2\pi\sigma^2}} \exp\left[ -\frac{|y - f(x)|^2}{2\sigma^2} \right].
\]
Fig. 2. Bayesian analysis of the product example: (a) posterior probability for the observed data $ab = 1$ for Gaussian observation noise of variance $\sigma^2 = 0.18$ and uniform prior probabilities over the plotted region, (b) cross section through the posterior at two different locations. Note the different thicknesses of the ridge; some local regions have more probability mass than others, even though the entire ridge has a constant maximum height.
\[ L(\tilde{x}, x) = -\delta(\tilde{x} - x). \]

\[ L(\tilde{x}, x) = |\tilde{x} - x|^2. \]

\[ L(\tilde{x}, x) = -\exp\left[-|K_L^{-1/2}(\tilde{x} - x)|^2\right], \]