Camera Models and Perspective Projection
Camera Models

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Cameras

- First photograph due to Niepce
- First on record shown in the book - 1822
- Basic abstraction is the pinhole camera
  - lenses required to ensure image is not too dark
  - various other abstractions can be applied
Pinhole cameras

- Abstract camera model - box with a small hole in it
- Pinhole cameras work in practice
Edgerton’s depiction of the reverse-reflection ‘peepshow’ of the Baptistry with which Brunelleschi demonstrated perspective in ~1420
The ‘Goldman’ Annunciation by Masolino (1424)

This complex architecture is estimated to have predated (and may have inspired) Masaccio’s ‘Trinity’ by four years.
The accurate perspective extends from the arch in the background to the coffers of the portico...
... and even details of the Virgin’s throne in the right foreground.
As required in accurate perspective, the 45° obliques project to distance points at the same height as the central vanishing point.
Viewpoint dependent Projections

Samuel von Hoogstraten
What’s wrong?

Ron Davis

*Six-Ninths Blue*, 1966

Ames Room
Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image or half-plane
- Polygons go to polygons
- Angles & distances not preserved

Degenerate cases:
- line through focal point yields point
- plane through focal point yields line
Geometric properties of projection

- Points go to points
- Lines go to lines
- Planes go to whole image
- Polygons go to polygons

Degenerate cases
- line through focal point to point
- plane through focal point to line
Distant objects are smaller
Parallel lines meet

Common to draw film plane in front of the focal point. Moving the film plane merely scales the image.
Polyhedra project to polygons

- (because lines project to lines)
Can we invert? Polygons -> Polyhedra?

- Although not unique, methods for interpreting line drawing exist.
- Key idea:
  - Classify junctions
  - Look for consistent sets of line labels, bounding polyhedra
  - “line labelling” - e.g.
    - Boundary lines
    - Corners (convex vertices)
  - Practical problem - too hard to extract lines and junctions to label from real images

Interpret drawing in terms of a set of planar facets--
Assign lines to facets
Determine facet normals
Curved surfaces are much more interesting under projection

- Crucial issue: outline is the set of points where the viewing direction is tangent to the surface
- This is a projection of a space curve, which varies from view to view of the surface
Vanishing points

- each set of parallel lines (=direction) meets at a different point
  - The vanishing point for this direction
- Sets of parallel lines on the same plane lead to collinear vanishing points.
  - The line is called the horizon for that plane

- Good ways to spot faked images
  - scale and perspective don’t work
  - vanishing points behave badly
  - supermarket tabloids are a great source.
Faked Photos
One Point

Two Point

Three Point

Brunelleschi's view of the Baptistry, according to Kemp (1990)
The equation of projection

• If you know $P=(x,y,z)$ and $f'$, what is $P'$?
  – Where does a point in view space end up on the screen?
The equation of projection

- Cartesian coordinates:
  - We have, by similar triangles, that
    \((x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z}, -f)\)

  - Ignore the third coordinate, and get
    \((x, y, z) \rightarrow (f \frac{x}{z}, f \frac{y}{z})\)
Homogenous coordinates

- Add an extra coordinate and use an equivalence relation
  - for 2D
    - equivalence relation \( k*(X,Y,Z) \) is the same as \( (X,Y,Z) \)
  - for 3D
    - equivalence relation \( k*(X,Y,Z,T) \) is the same as \( (X,Y,Z,T) \)

- Basic notion
  - Possible to represent points “at infinity”
    - Where parallel lines intersect
    - Where parallel planes intersect
  - Possible to write the action of a perspective camera as a matrix
The camera matrix

- Turn previous expression into HC’s
  - HC’s for 3D point are \((X,Y,Z,T)\)
  - HC’s for point in image are \((U,V,W)\)

Relation between HC’s and image coordinates

\[
\begin{pmatrix}
U \\
V \\
W \\
\end{pmatrix} = \begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 1/f & 0 \\
\end{pmatrix} \begin{pmatrix}
X \\
Y \\
Z \\
T \\
\end{pmatrix}
\]

\(x_s = U/W\)

\(y_s = U/W\)
Weak perspective

• Issue
  – perspective effects only large with depth variation-
  – Perspective effects frequently small over the scale of individual objects
  – collect points into a group at about the same depth, then divide each point by the depth of its group
  – Adv: easy
  – Disadv: wrong
Orthographic projection
Orthographic via Telecentric Lens
The projection matrix for orthographic projection

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = 
\begin{pmatrix}
1 & 0 & 0 & 0 \\
0 & 1 & 0 & 0 \\
0 & 0 & 0 & 1
\end{pmatrix}
\begin{pmatrix}
X \\
Y \\
Z \\
T
\end{pmatrix}
\]
The reason for lenses
Pinhole too big - many directions are averaged, blurring the image

Pinhole too small - diffraction effects blur the image

Generally, pinhole cameras are *dark*, because a very small set of rays from a particular point hits the screen.
The thin lens

\[ \frac{1}{z'} - \frac{1}{z} = \frac{1}{f} \]

\[ f = \frac{R}{2(n-1)} \]

\[ z' = \frac{zf}{z+f} \]
Is limited focus bad?

1) Know your current focal length
2) Blur gradient => Depth
Spherical aberration
Lens systems
Vignetting
Other (possibly annoying) phenomena

• Chromatic aberration
  – Light at different wavelengths follows different paths; hence, some wavelengths are defocussed
  – Machines: coat the lens
  – Humans: live with it

• Scattering at the lens surface
  – Some light entering the lens system is reflected off each surface it encounters (Fresnel’s law gives details)
  – Machines: coat the lens, interior
  – Humans: live with it (various scattering phenomena are visible in the human eye)

• Geometric phenomena (Barrel distortion, etc.)
Human Eye-thick lens
CCD Cameras

Noise sources:
1) Spatial response/Efficiency
2) Thermal fluctuations
3) Shot Noise
4) Read Noise (amplifier)
5) Quantization

\[
I = \int_t \int_\lambda \int_y \int_x E(x, y, \lambda, t)S(x, y)q(\lambda)dx dy d\lambda dt,
\]
Camera parameters

• Issue
  – camera may not be at the origin, looking down the z-axis
    • extrinsic parameters
  – one unit in camera coordinates may not be the same as one unit in world coordinates
    • intrinsic parameters - focal length, principal point, aspect ratio, angle between axes, etc.

\[
\begin{pmatrix}
U \\
V \\
W
\end{pmatrix} = \begin{pmatrix}
\text{Transformation} & 1 & 0 & 0 & 0 \\
\text{representing} & 0 & 1 & 0 & 0 \\
\text{intrinsic parameters} & 0 & 0 & 1 & 0 \\
\end{pmatrix} \begin{pmatrix}
\text{Transformation} & X \\
\text{representing} & Y \\
\text{extrinsic parameters} & Z \\
\end{pmatrix}
\]
Camera calibration

• **Issues:**
  – what are intrinsic parameters of the camera?
  – what is the camera matrix? (intrinsic+extrinsic)

• **General strategy:**
  – view calibration object
  – identify image points
  – obtain camera matrix by minimizing error
  – obtain intrinsic parameters from camera matrix

• **Error minimization:**
  – Linear least squares
    • easy problem numerically
    • solution can be rather bad
  – Minimize image distance
    • more difficult numerical problem
    • solution usually rather good,
      • start with linear least squares
  – Numerical scaling is an issue