Unsupervised Segmentation

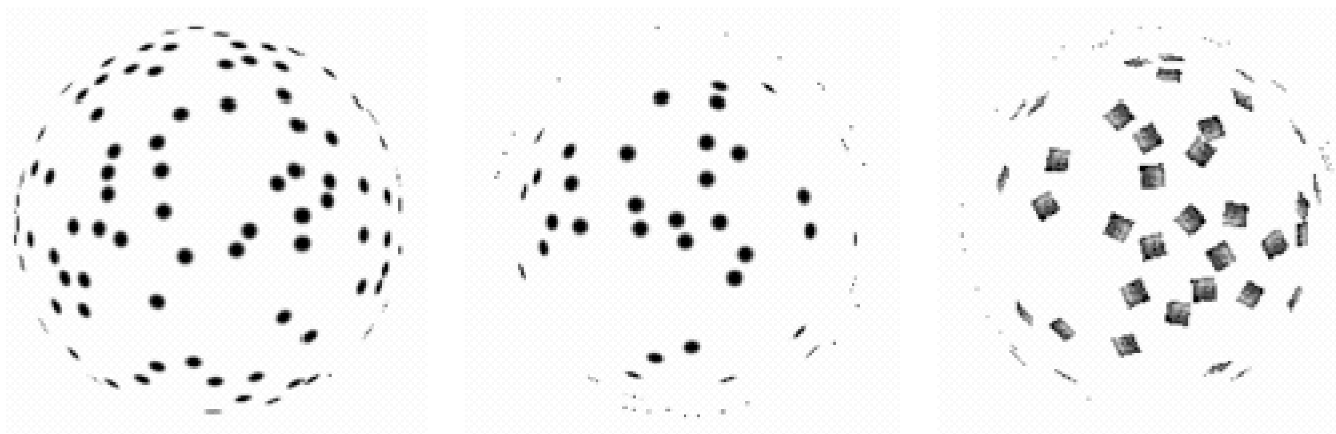
Unsupervised Segmentation and Grouping

- Motivation: Many computer vision problems would be easy, except for background interference.
- Unsupervised Segmentation: no training data
- Use: Obtain a compact representation from an image/motion sequence/set of tokens
- Should support application
- Broad theory is absent at present

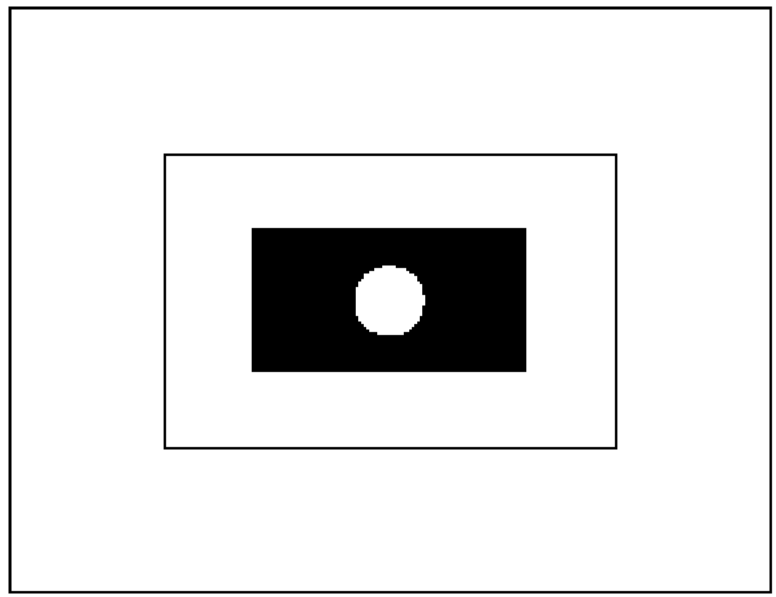
- Grouping (or clustering)
  - collect together tokens that "belong together"
- Fitting
  - associate a model with tokens
  - issues
    - which model?
    - which token goes to which element?
    - how many elements in the model?

General ideas

- Features (tokens)
  - whatever we need to group (pixels, points, surface elements, etc., etc.)
- top down segmentation (model based)
  - features belong together because they lie on the same object.
- bottom up segmentation (image based)
  - features belong together because they are locally coherent
- These two are not mutually exclusive

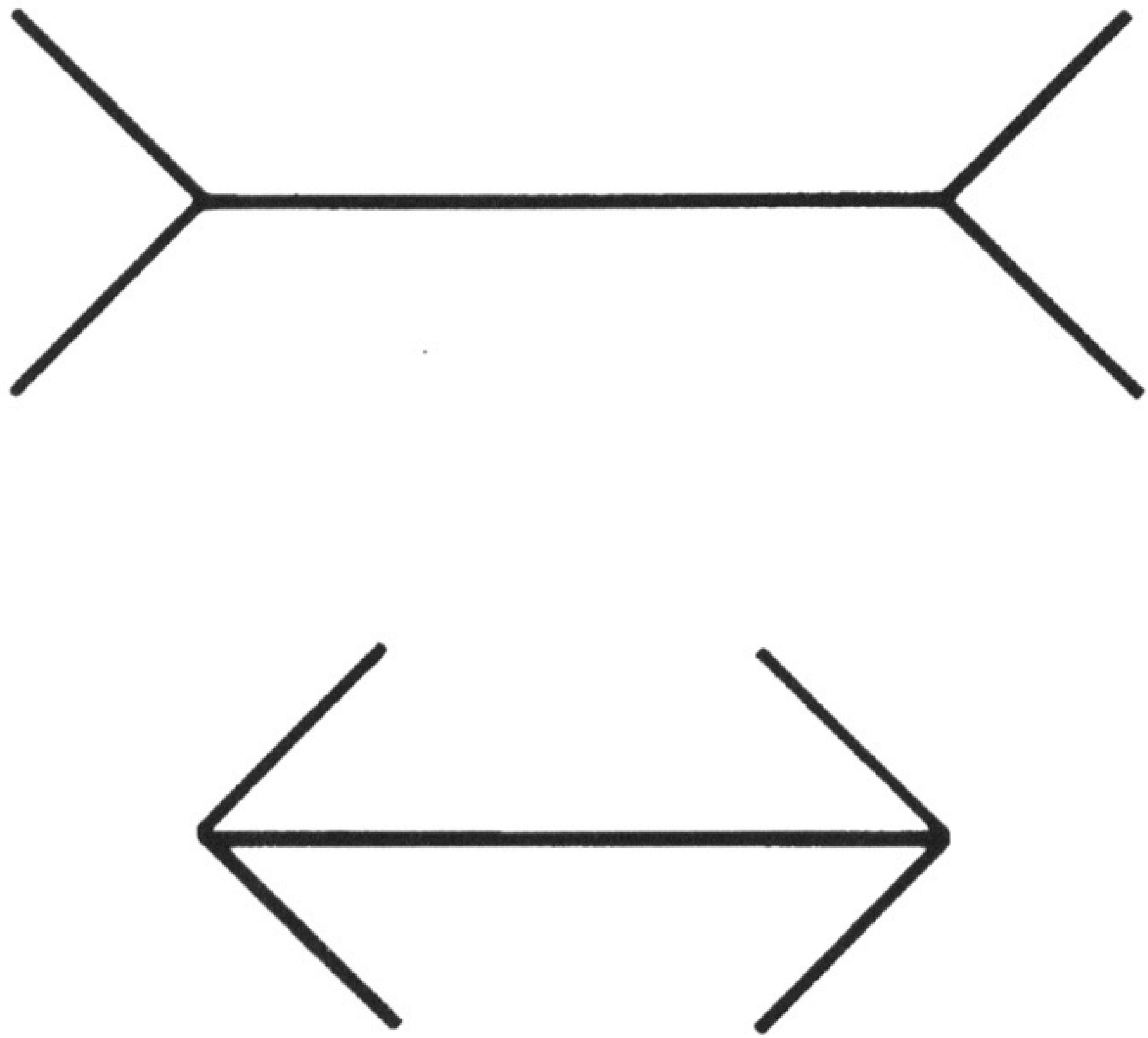


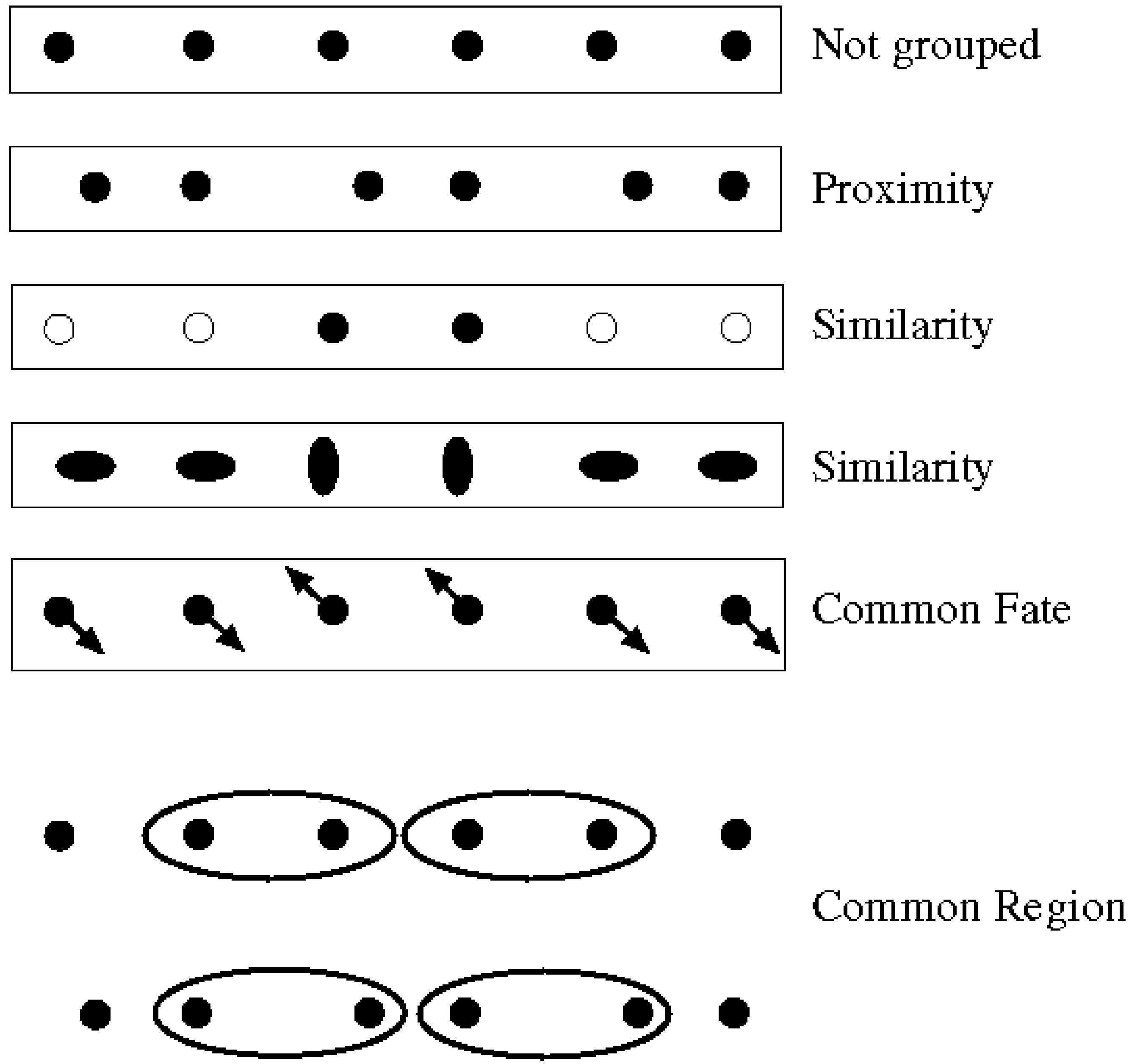
Why do these features belong together?

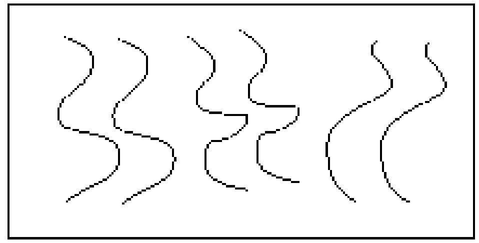


Basic ideas of grouping in humans

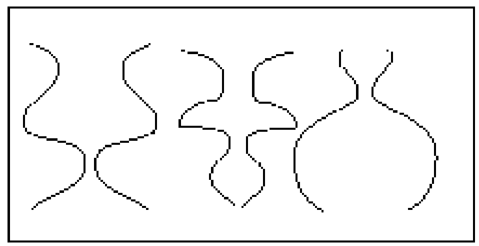
- Figure-ground discrimination
  - grouping can be seen in terms of allocating some elements to a figure, some to ground
  - impoverished theory
- Gestalt properties
  - elements in a collection of elements can have properties that result from relationships (Muller-Lyer effect)
    - gestaltqualitat
  - A series of factors affect whether elements should be grouped together
    - Gestalt factors



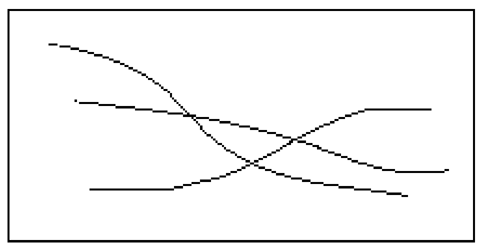

Not grouped
Proximity
Similarity
Similarity
Common Fate
Common Region



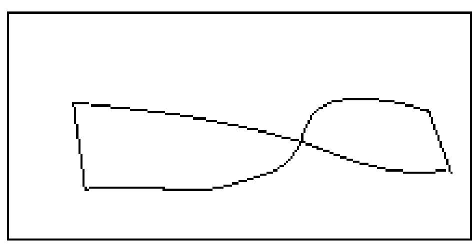
Parallelism



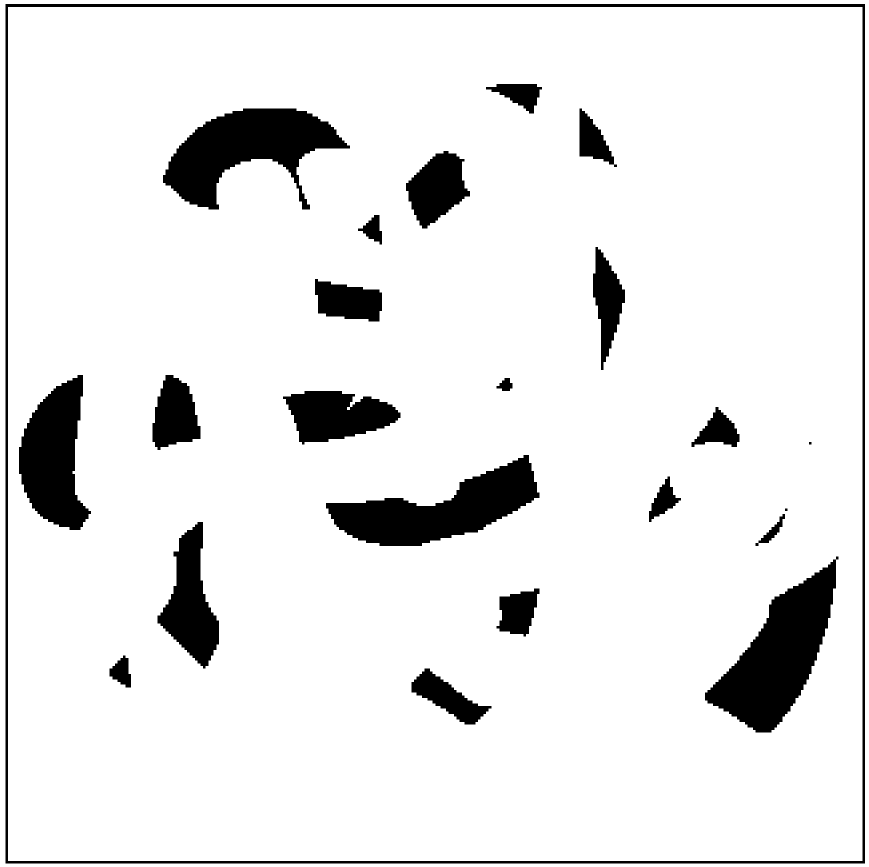
Symmetry

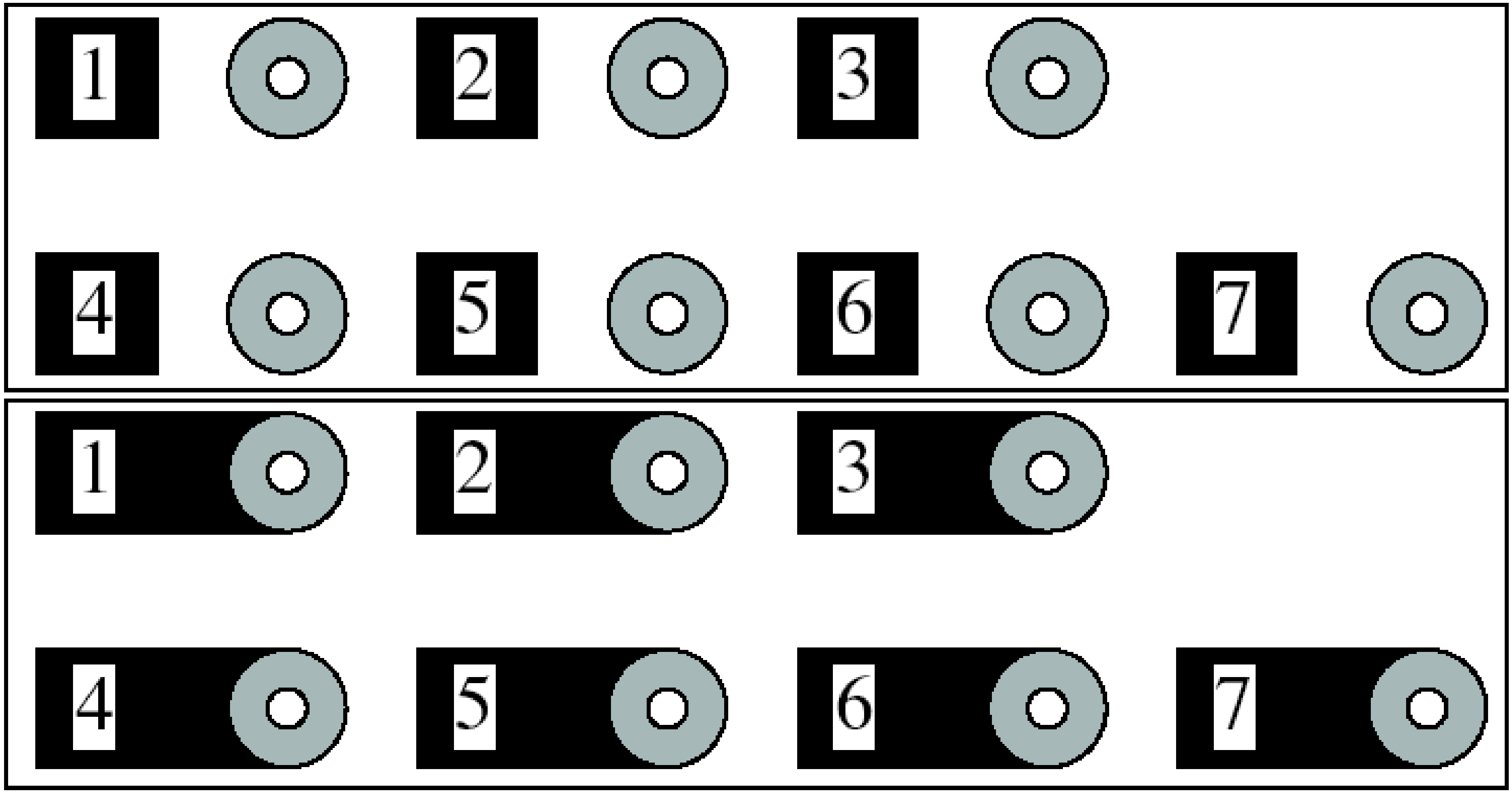


Continuity



Closure




1
2
3
4
5
6
7
1
2
3
4
5
6
7

Are Gestalt laws the result of observed regularity in scenes?

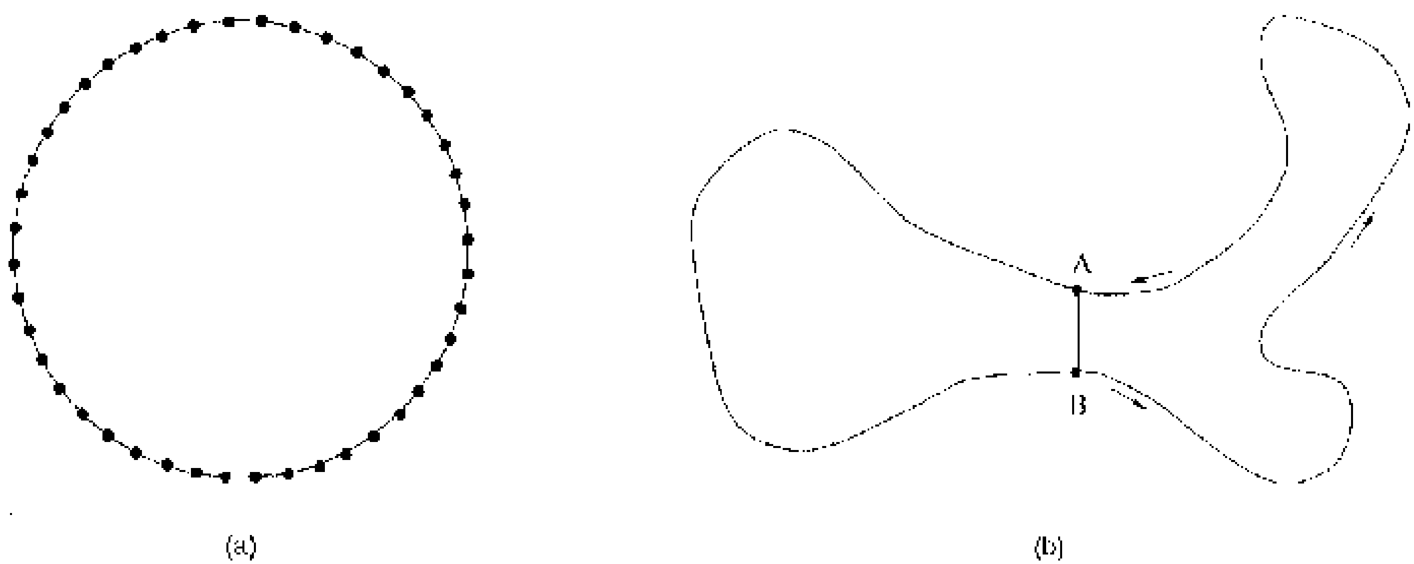
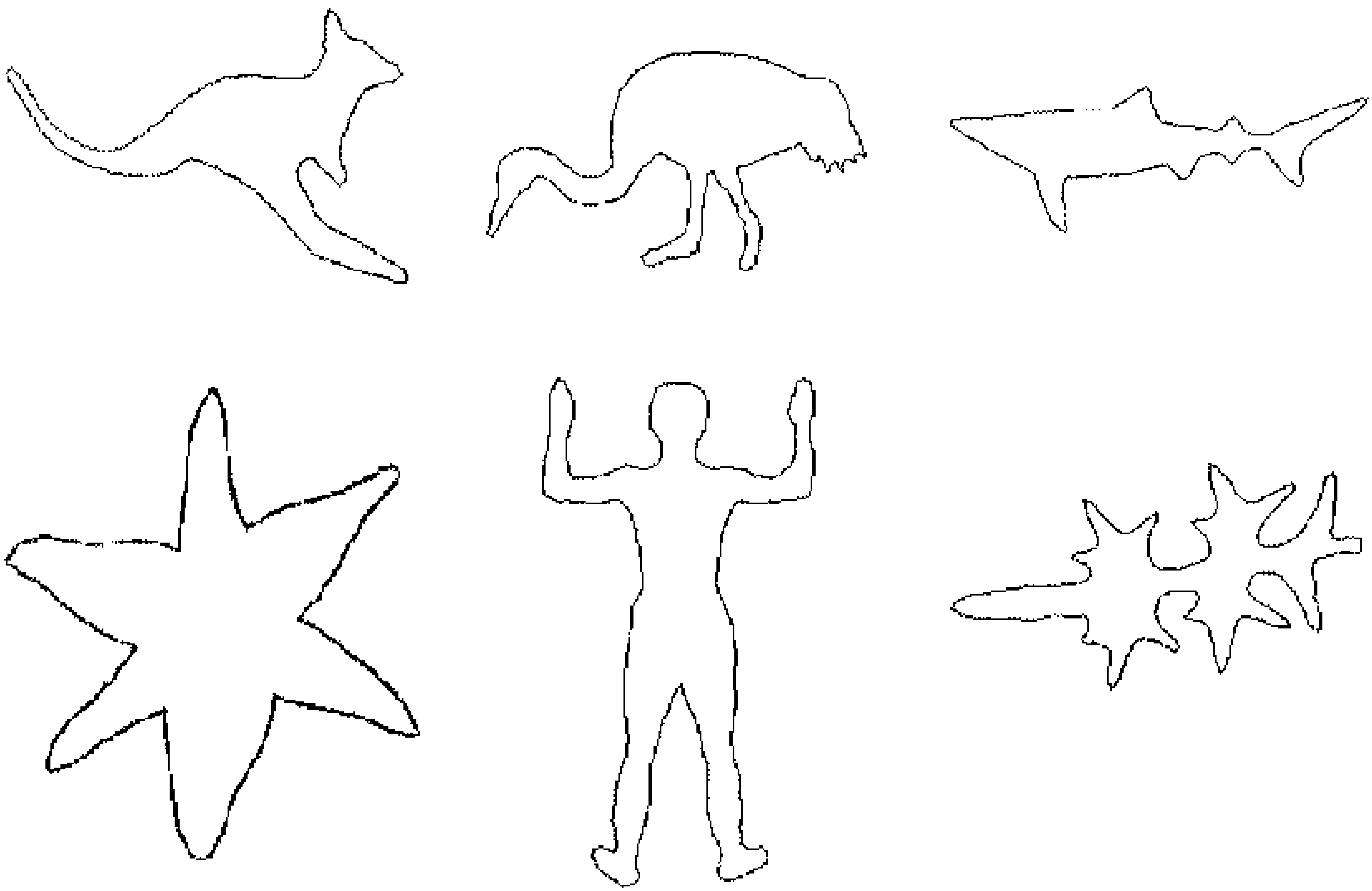
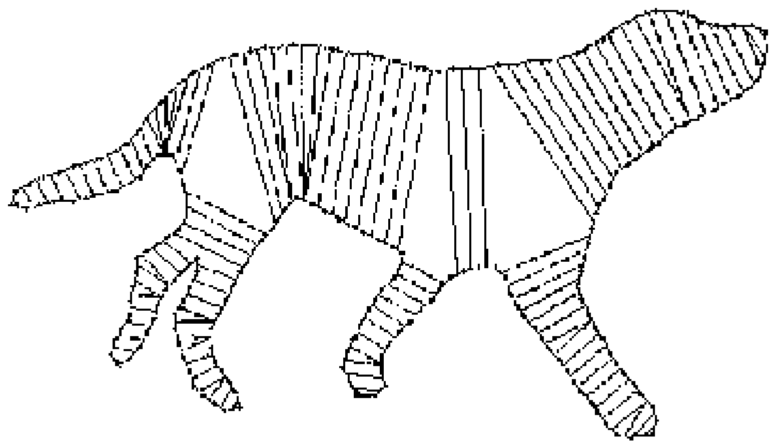
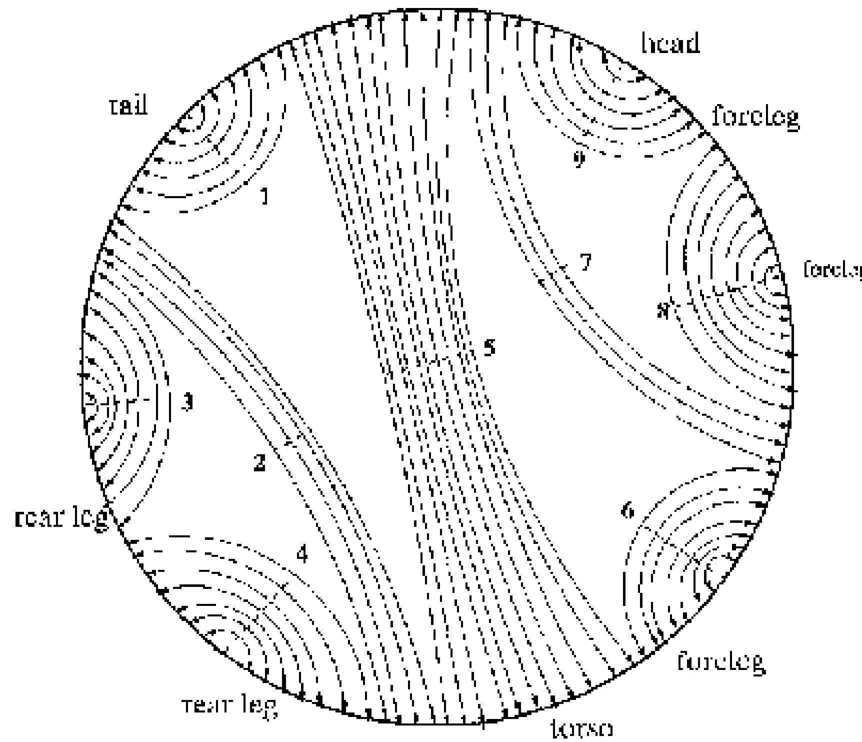


Fig. 1. (a) A 1D Markov random field where the nodes represent random variables for positions of contour points. (b) Node A is spatially adjacent to point B, but it is far away from B in the circular neighborhood of (a).





(a)


head
tail
foreleg
9
1
7
foreleg
8
5
3
2
rear leg
6
4
foreleg
rear leg
torso

(b)

Collect statistics on local curvature and adjacency for natural contours.

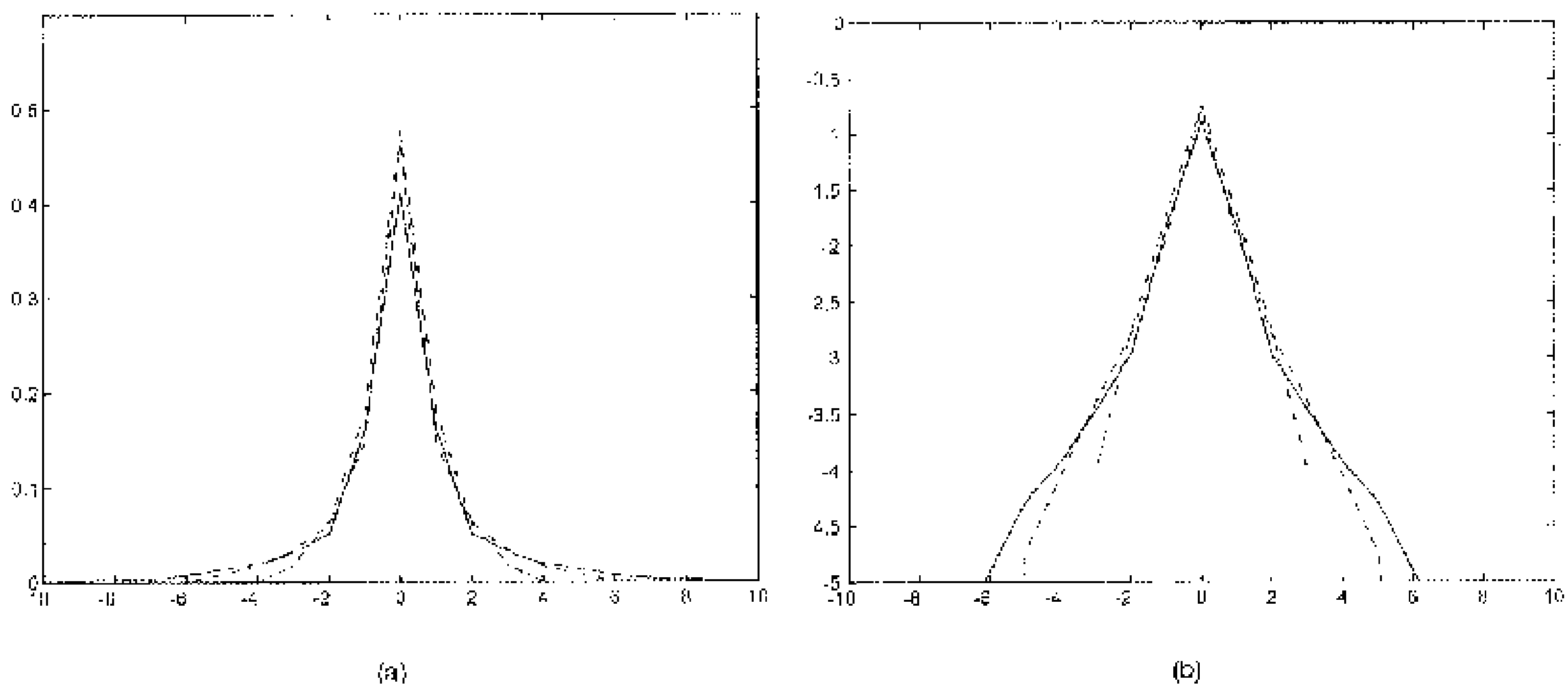


Fig. 5. (a) The histograms of $\kappa(s)$ averaged over 22 animate objects at scale 0 (solid curve), scale 1 (dashed curve), and scale 2 (dash-dotted curve), the horizontal axis is $\kappa(s)$ with unit $d\kappa =$ [illegible]. (b) The logarithm of curves in (a).

Sampled boundaries from learning probability model.

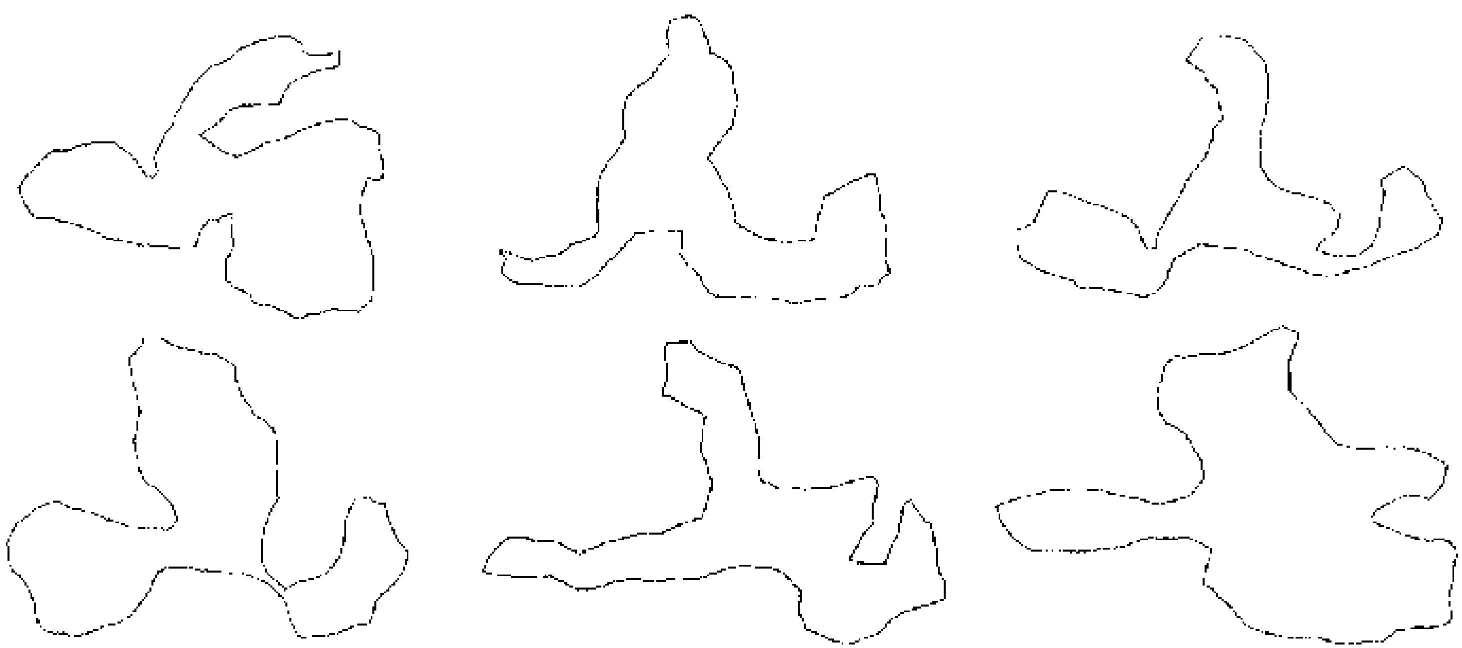


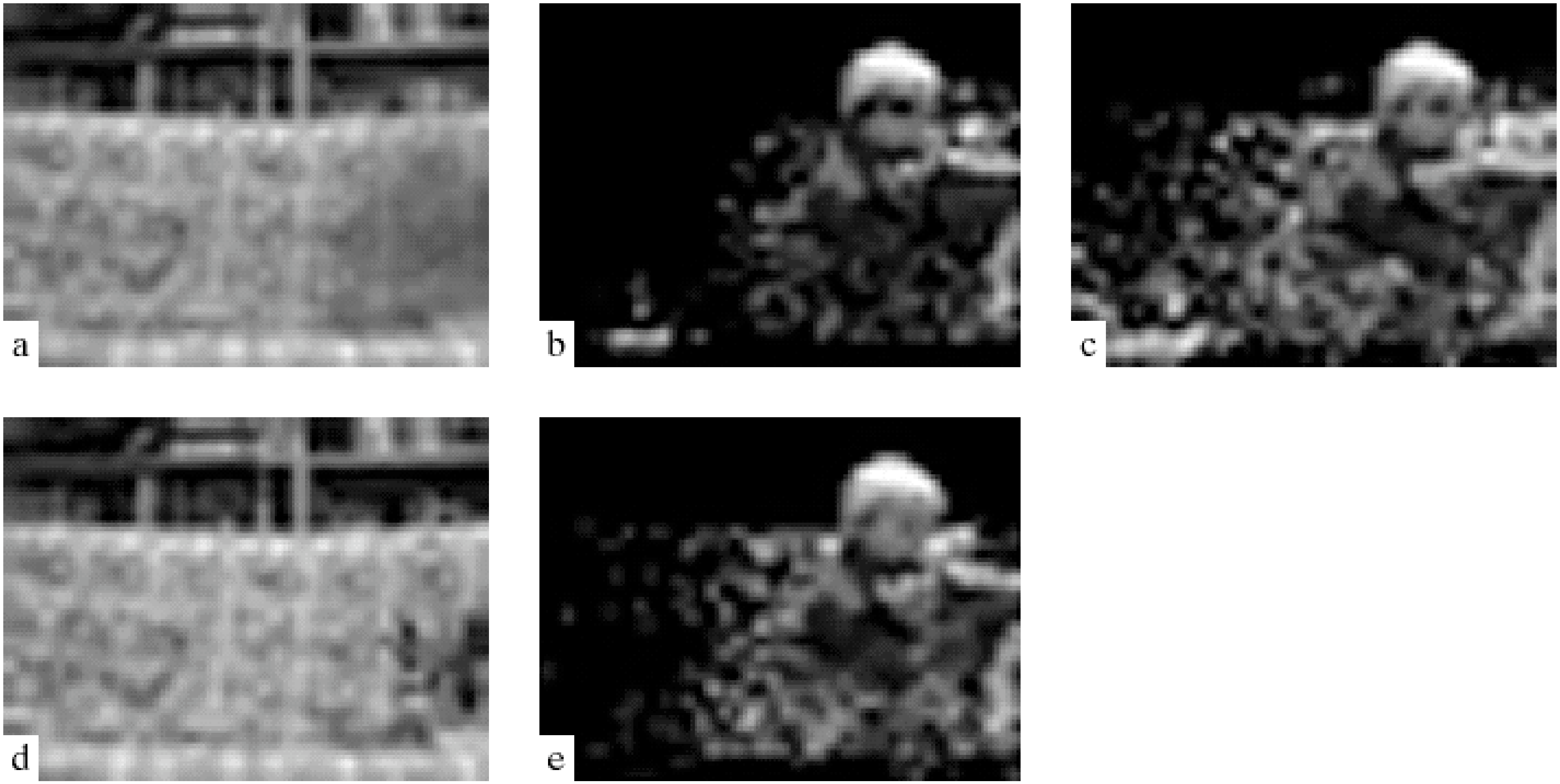
Fig. 11. Six of the synthesized shapes with curvature histogram matched to animate shapes, $\mu^{(i)}_{syn} = \mu^{(i)}_{obs}$. The histograms of these synthesized shapes are shown by the dashed curves in Fig. 12.

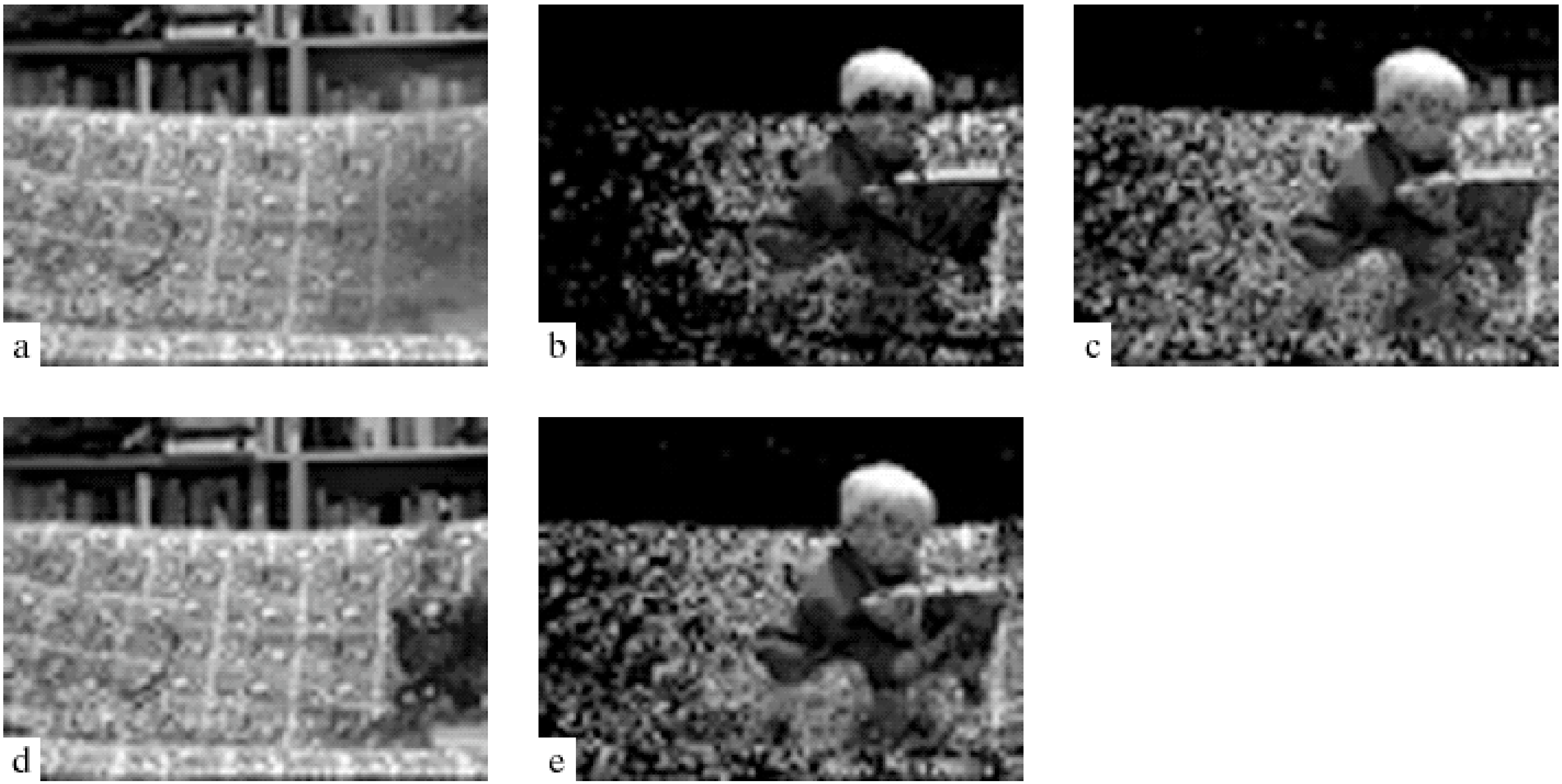
Technique: Shot Boundary Detection

- Find the shots in a sequence of video
  - shot boundaries usually result in big differences between succeeding frames
- Strategy:
  - compute interframe distances
  - declare a boundary where these are big
- Possible distances
  - frame differences
  - histogram differences
  - block comparisons
  - edge differences
- Applications:
  - representation for movies, or video sequences
    - find shot boundaries
    - obtain “most representative” frame
  - supports search

Technique: Background Subtraction

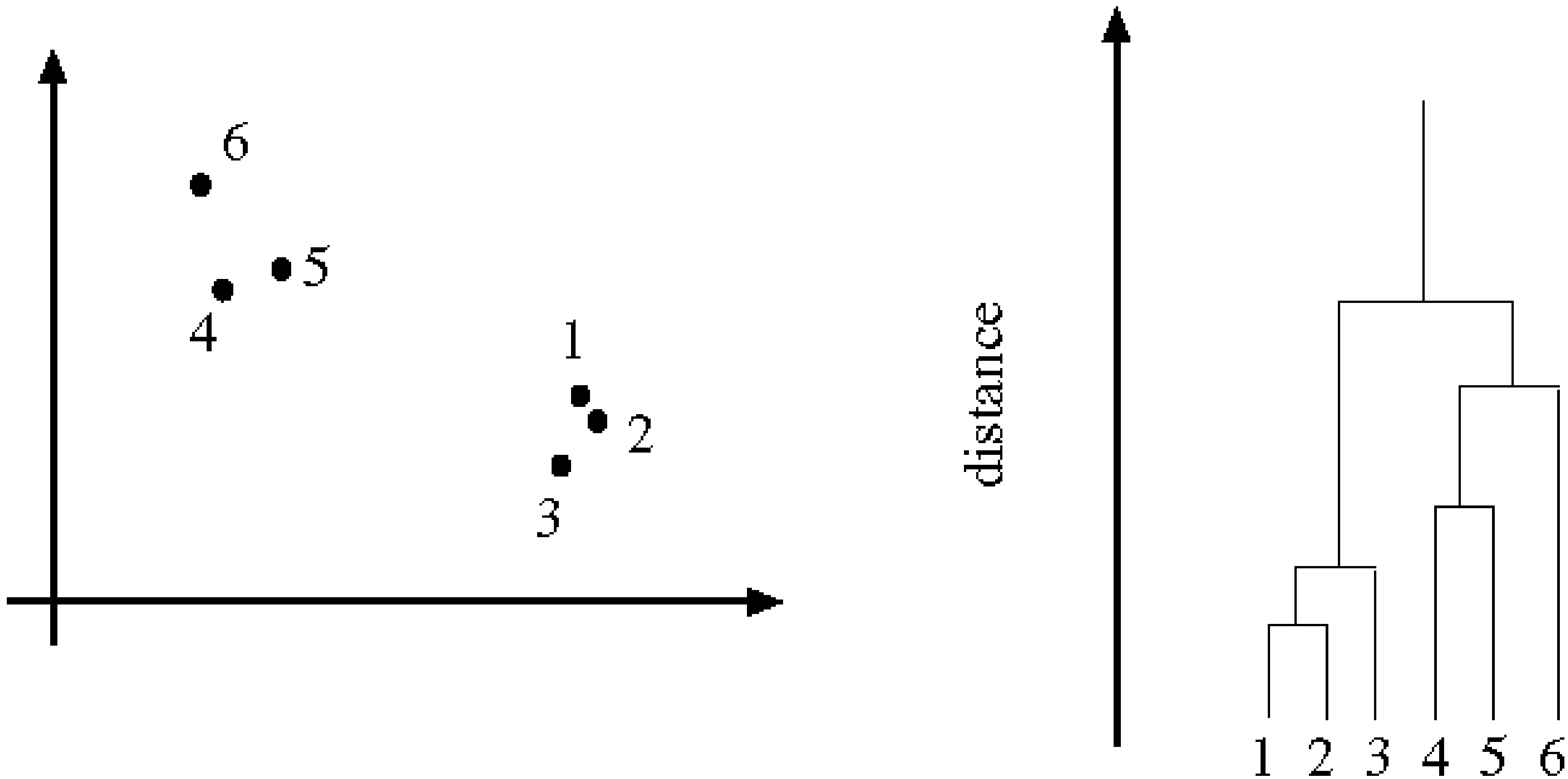
- If we know what the background looks like, it is easy to identify “interesting bits”
- Applications
  - Person in an office
  - Tracking cars on a road
  - surveillance
- Approach:
  - use a moving average to estimate background image
  - subtract from current frame
  - large absolute values are interesting pixels
    - trick: use morphological operations to clean up pixels


a
b
c
d
e


a
b
c
d
e

Segmentation as clustering

- Cluster together (pixels, tokens, etc.) that belong together
- Agglomerative clustering
  - attach closest to cluster it is closest to
  - repeat
- Divisive clustering
  - split cluster along best boundary
  - Repeat
- Point-Cluster distance
  - single-link clustering
  - complete-link clustering
  - group-average clustering
- Dendrograms
  - yield a picture of output as clustering process continues


6
5
4
1
2
3
distance
1 2 3 4 5 6

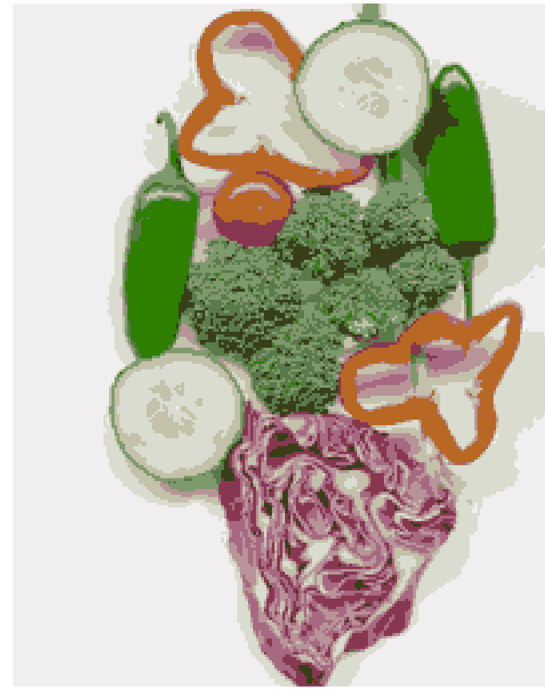
K-Means

- Choose a fixed number of clusters
- Choose cluster centers and point-cluster allocations to minimize error
- can't do this by search, because there are too many possible allocations.
- Algorithm
  - fix cluster centers; allocate points to closest cluster
  - fix allocation; compute best cluster centers
- x could be any set of features for which we can compute a distance (careful about scaling)

$$\sum_{i \in \text{clusters}} \left\{ \sum_{j \in \text{elements of i'th cluster}} \left\| x_j - \mu_i \right\|^2 \right\}$$

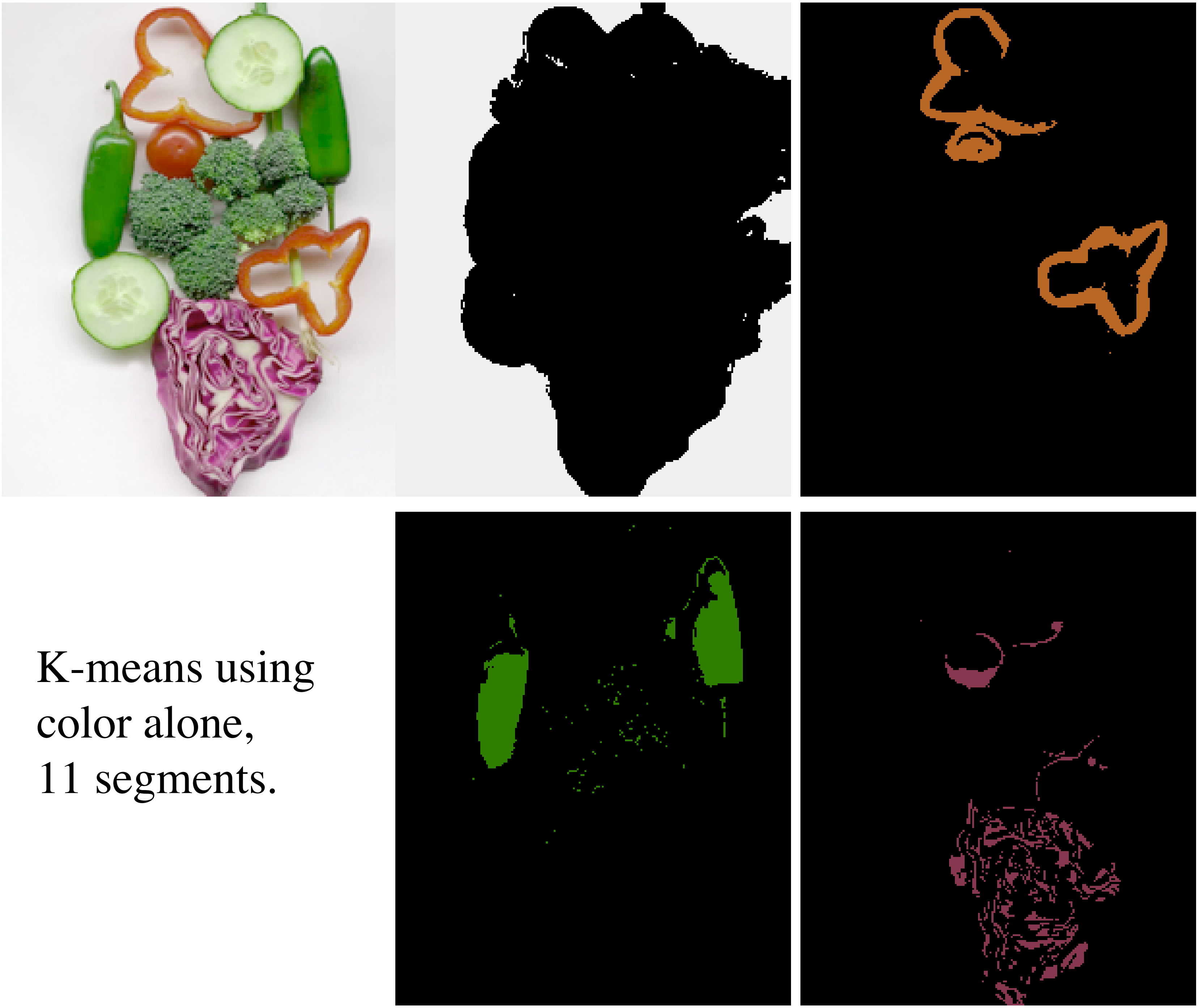
K-means clustering using intensity alone and color alone

Image

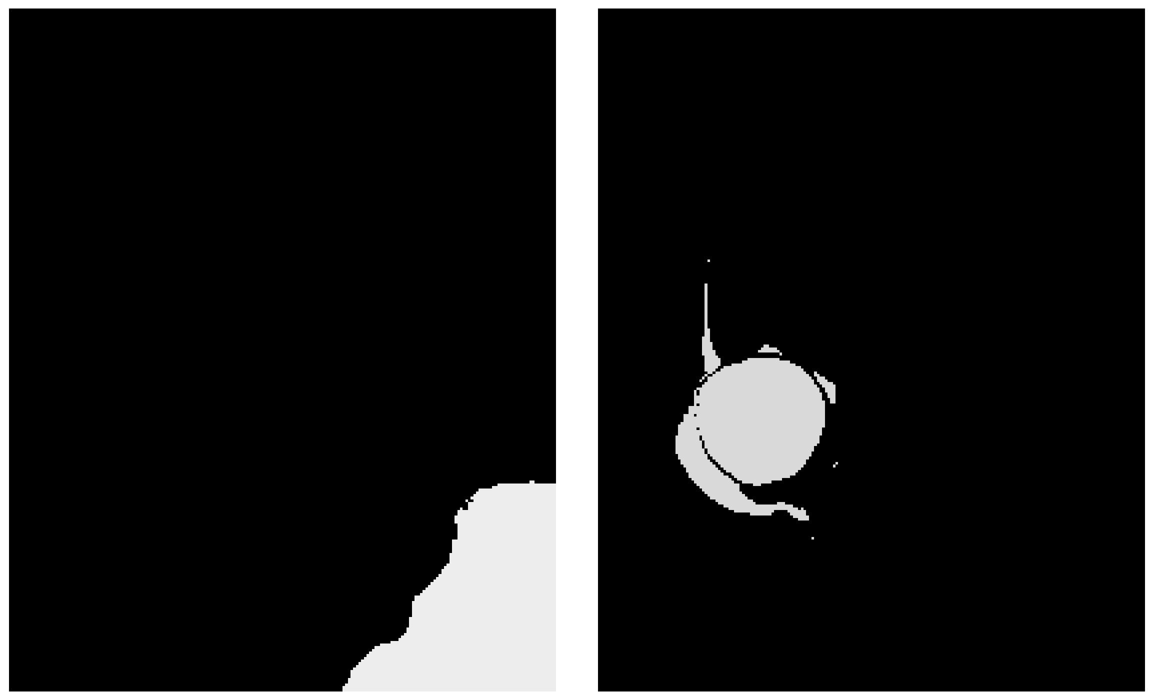


Clusters on color

K-means using color alone, 11 segments



K-means using
color alone,
11 segments.

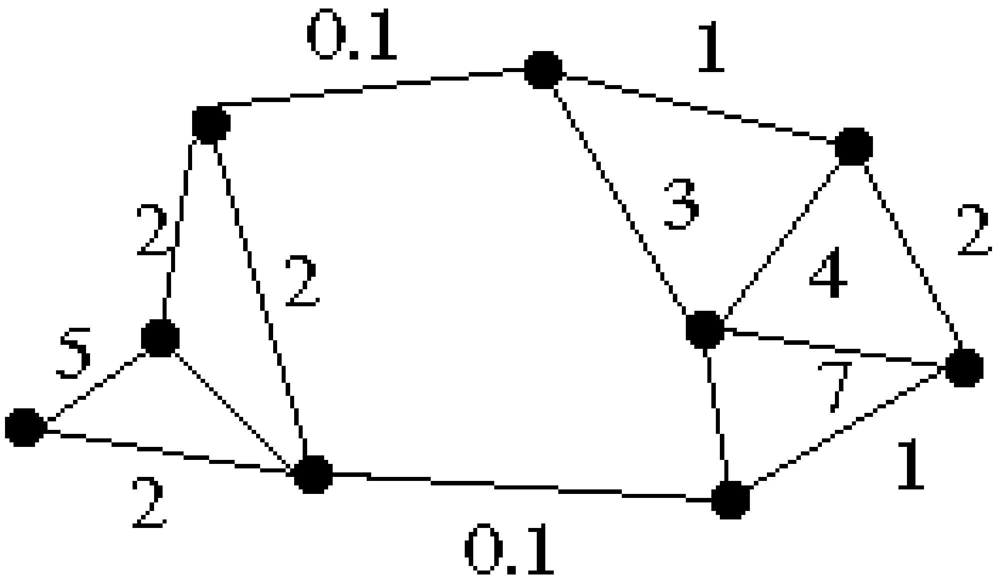


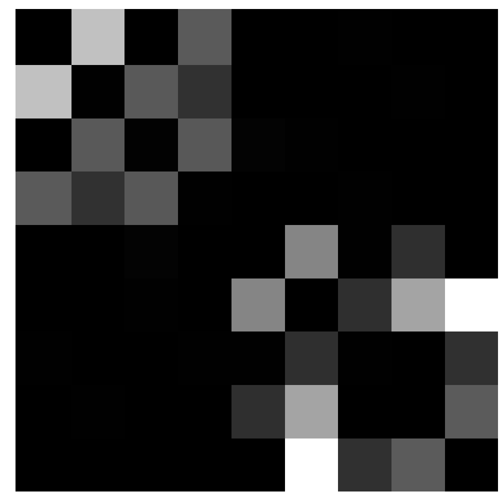
K-means using colour and position, 20 segments

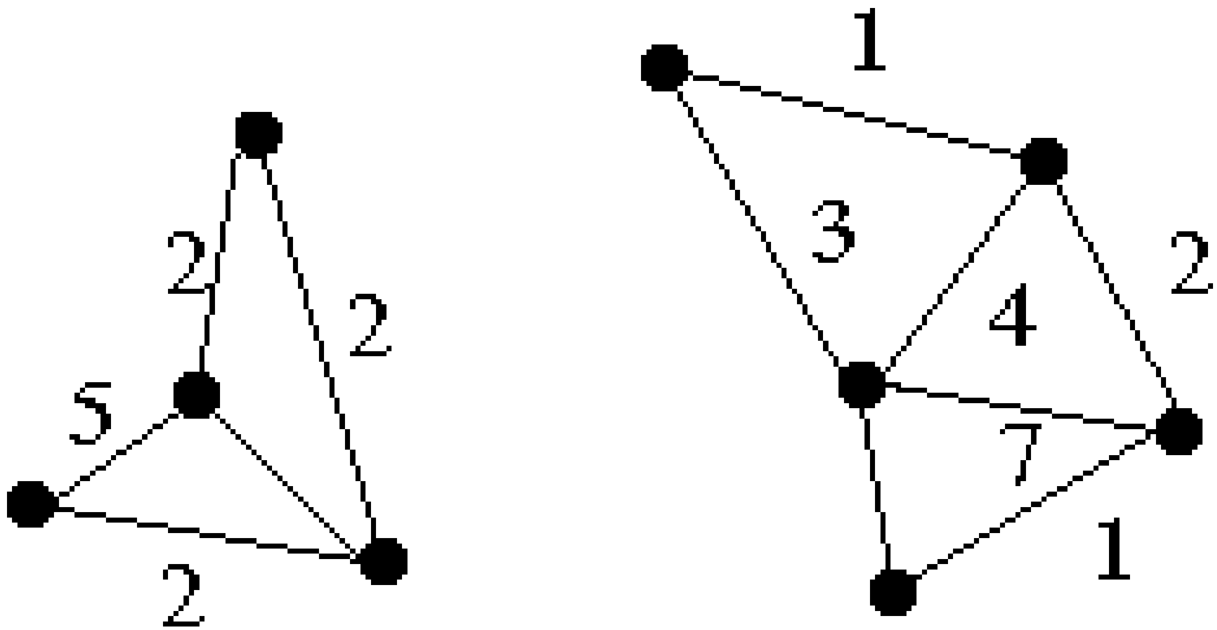
Graph theoretic clustering

- Represent tokens using a weighted graph.
  - affinity matrix
- Cut up this graph to get subgraphs with strong interior links

Image Segmentation as Graph Partitioning


0.1
1
2
2
3
2
4
5
7
2
1
0.1




2
2
5
2
1
3
4
2
7
1

Boundaries of image regions defined by a number of attributes

- Brightness/color
- Texture
- Motion
- Stereoscopic depth
- Familiar configuration

Measuring Affinity

Intensity

$$aff(x,y) = \exp\left\{ -\left(\frac{1}{2\sigma_i^2} \right) \left(\| I(x) - I(y) \|^2 \right) \right\}$$

Distance

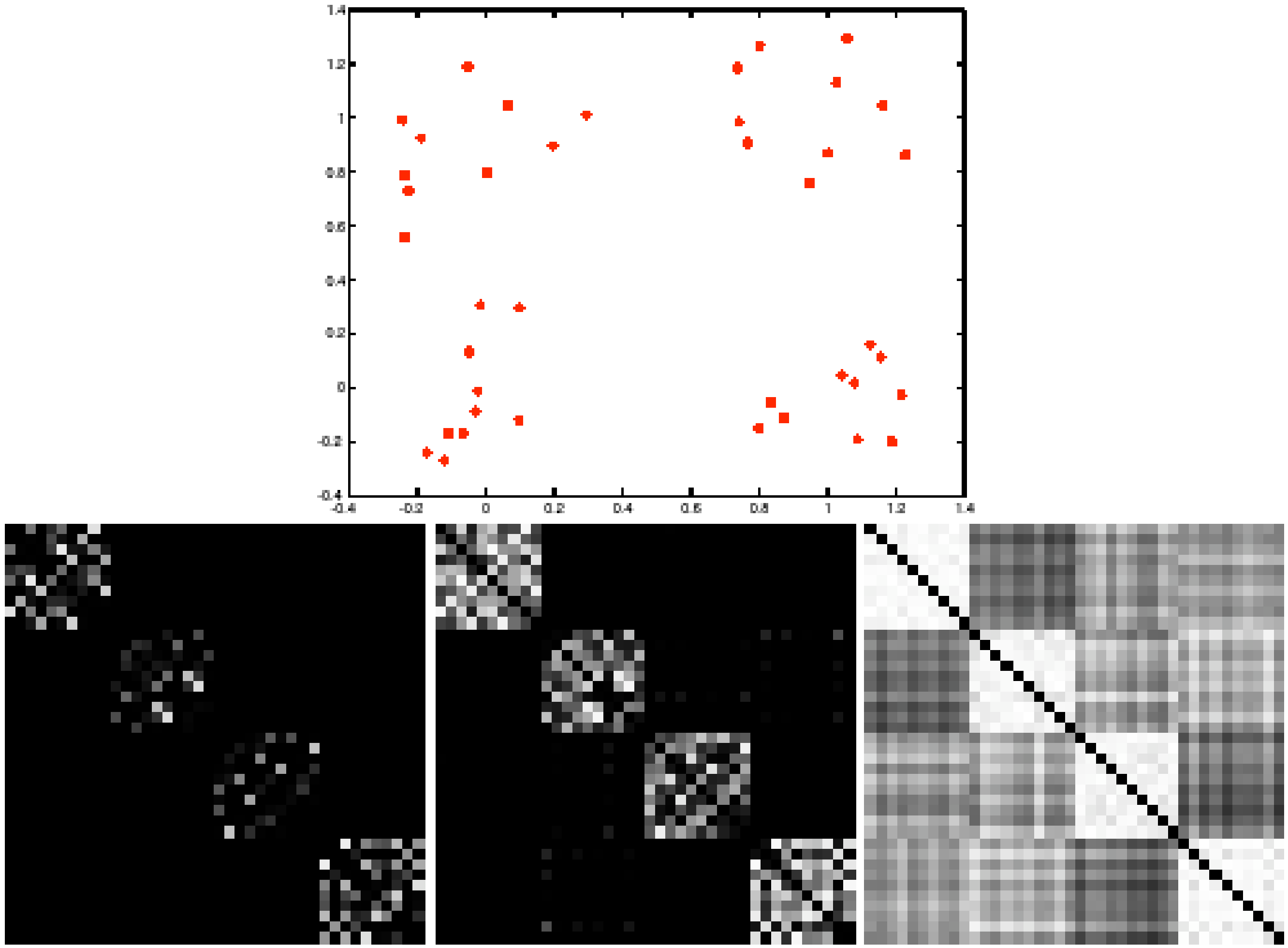
$$aff(x,y) = \exp\left\{ -\left(\frac{1}{2\sigma_d^2} \right) \left(\| x - y \|^2 \right) \right\}$$

Texture

$$aff(x,y) = \exp\left\{ -\left(\frac{1}{2\sigma_t^2} \right) \left(\| c(x) - c(y) \|^2 \right) \right\}$$

c(x) denotes a histogram,for instance

Scale affects affinity



Eigenvectors and cuts

- Simplest idea: we want a vector a giving the association between each element and a cluster
- We want elements within this cluster to, on the whole, have strong affinity with one another

$Let \quad w_{ij} = aff(x_i, x_j)$ for pixels x_i & x_j

- We could maximize

$$E = a^T W a$$

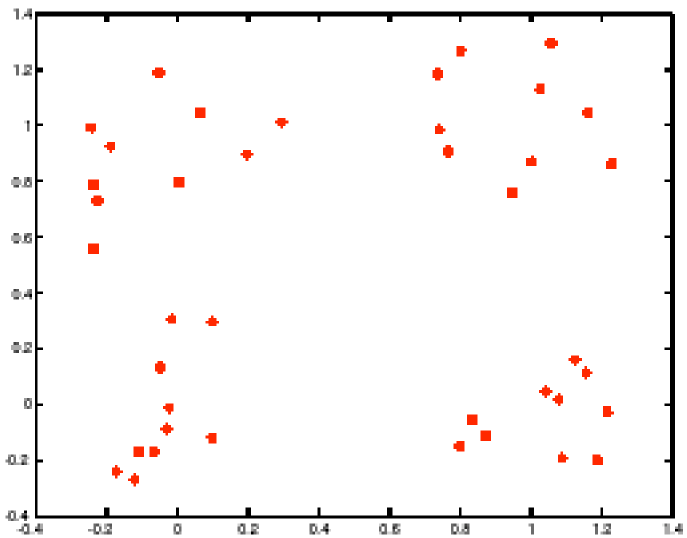
- But need the constraint

$$a^T a = 1$$

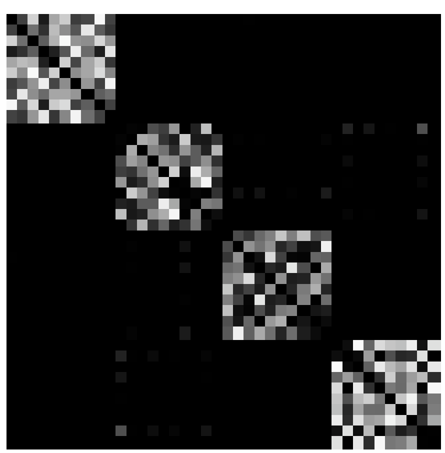
- This is an eigenvalue problem - choose the eigenvector of A with largest eigenvalue

$$\begin{aligned}
&Maximize: \\
&E = a^T W a + \lambda(1 - a^T a) \\
&\text{Set}: \frac{\partial E}{\partial a} = 0 \\
&\qquad = 2Wa - 2\lambda a \\
&\Rightarrow \; Wa = \lambda a
\end{aligned}$$

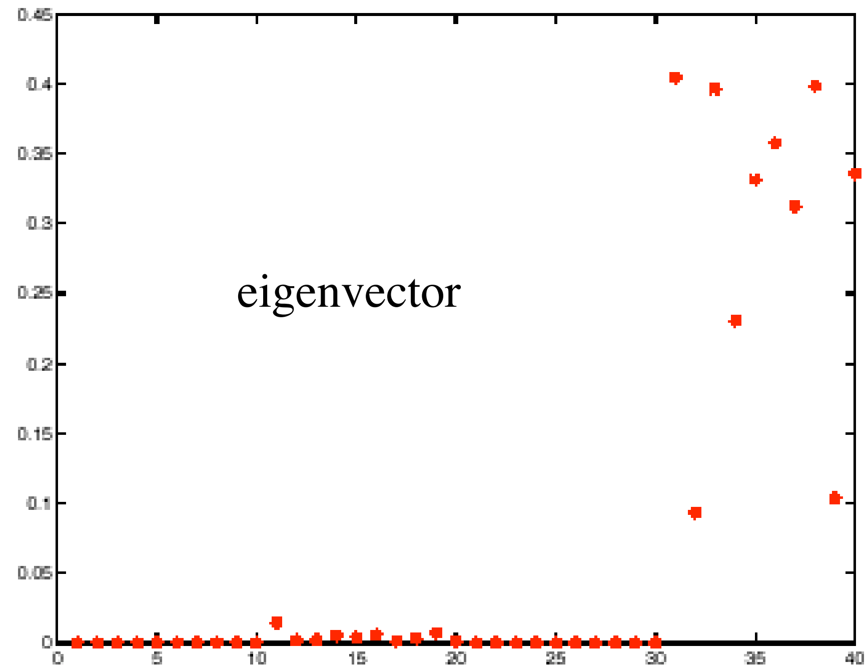
Example eigenvector



points



matrix



More than two segments

- Two options
  - Recursively split each side to get a tree, continuing till the eigenvalues are too small
  - Use the other eigenvectors

Normalized cuts

- Current criterion evaluates within cluster similarity, but not across cluster difference
- Instead, we'd like to maximize the within cluster similarity compared to the across cluster difference
- Write graph as V, one cluster as A and the other as B

$$assoc(A,B) = \sum_{i \in A, j \in B} W(i,j)$$

- Maximize

$$Nassoc(A,B) = \left(\frac{assoc(A,A)}{assoc(A,V)}\right) + \left(\frac{assoc(B,B)}{assoc(B,V)}\right)$$

- i.e. construct A, B such that their within cluster similarity is high compared to their association with the rest of the graph

Normalized cuts

- Write a vector y whose elements are 1 if item is in A, -b if it's in B
- Write the matrix of the graph as W, and the matrix which has the row sums of W on its diagonal as D, 1 is the vector with all ones.
- Criterion becomes

$$\min_y \left(\frac{y^T (D - W) y}{y^T D y} \right)$$

- and we have a constraint

$$y^T D 1 = 0$$

- This is hard to do, because y's values are quantized

Normalized cuts

- Instead, solve the generalized eigenvalue problem

$$\max_y \left(y^T (D - W) y \right) \text{ subject to } \left(y^T D y = 1 \right)$$

- which gives

$$(D - W) y = \lambda D y$$

- Now look for a quantization threshold that maximises the criterion --- i.e all components of y above that threshold go to one, all below go to -b

Given a partition of nodes of a graph, V, into two sets A and B, let $\boldsymbol{x}$ be an $N = |\boldsymbol{V}|$ dimensional indicator vector, $x_i = 1$ if node i is in A, and -1 otherwise. Let $\boldsymbol{d}(\mathrm{i}) = \sum_j w(i,j)$, be the total connection from node $\boldsymbol{i}$ to all other nodes. With the definitions $\boldsymbol{x}$ and $\boldsymbol{d}$ we can rewrite $Ncut(A,B)$ as:

$$
\begin{aligned}
Ncut(A,B) &= \frac{cut(A,B)}{assoc(A,V)} + \frac{cut(B,A)}{assoc(B,V)} \\
&= \frac{\sum_{(\boldsymbol{x}_i>0,\boldsymbol{x}_j<0)} -w_{ij}\boldsymbol{x}_i\boldsymbol{x}_j}{\sum_{\boldsymbol{x}_i>0}\boldsymbol{d}_i} \\
&\quad + \frac{\sum_{(\boldsymbol{x}_i<0,\boldsymbol{x}_j>0)} -w_{ij}\boldsymbol{x}_i\boldsymbol{x}_j}{\sum_{\boldsymbol{x}_i<0}\boldsymbol{d}_i}
\end{aligned}
$$

Let $\mathbf{D}$ be an $N \times N$ diagonal matrix with $\boldsymbol{d}$ on its diagonal, $\mathbf{W}$ be an $N \times N$ symmetrical matrix with $\mathrm{W(i,j)} = w_{ij}$, $k = \frac{\sum_{x_i>0}\boldsymbol{d}_i}{\sum_i \boldsymbol{d}_i}$, and $\mathbf{1}$ be an $N \times 1$ vector of all ones. Using the fact $\frac{\mathbf{1}+\boldsymbol{x}}{2}$ and $\frac{\mathbf{1}-\boldsymbol{x}}{2}$ are indicator vectors for $x_i > 0$ and $x_i < 0$ respectively, we can rewrite $4[Ncut(\boldsymbol{x})]$ as:

$$
\begin{aligned}
&= \frac{(\mathbf{1}+\boldsymbol{x})^T(\mathbf{D}-\mathbf{W})(\mathbf{1}+\boldsymbol{x})}{k\mathbf{1}^T\mathbf{D}\mathbf{1}} + \frac{(\mathbf{1}-\boldsymbol{x})^T(\mathbf{D}-\mathbf{W})(\mathbf{1}-\boldsymbol{x})}{(1-k)\mathbf{1}^T\mathbf{D}\mathbf{1}} \\
&= \frac{(\boldsymbol{x}^T(\mathbf{D}-\mathbf{W})\boldsymbol{x}+\mathbf{1}^T(\mathbf{D}-\mathbf{W})\mathbf{1})}{k(1-k)\mathbf{1}^T\mathbf{D}\mathbf{1}} + \frac{2(1-2k)\mathbf{1}^T(\mathbf{D}-\mathbf{W})\boldsymbol{x}}{k(1-k)\mathbf{1}^T\mathbf{D}\mathbf{1}}
\end{aligned}
$$

$$b = \frac{k}{1-k},$$

Setting $\boldsymbol{y} = (\mathbf{1} + \boldsymbol{x}) - b(\mathbf{1} - \boldsymbol{x})$,

Shi & Malik(1997) show:

$$min_{\boldsymbol{x}} Ncut(\boldsymbol{x}) = min_{\boldsymbol{y}} \frac{\boldsymbol{y}^T(\mathbf{D} - \mathbf{W})\boldsymbol{y}}{\boldsymbol{y}^T \mathbf{D} \boldsymbol{y}},$$

with the condition $\boldsymbol{y}(i) \in \{1, -b\}$ and $\boldsymbol{y}^T \mathbf{D} \mathbf{1} = 0$.

This is Equivalent to:

$$\mathbf{D}^{-\frac{1}{2}}(\mathbf{D} - \mathbf{W})\mathbf{D}^{-\frac{1}{2}}\boldsymbol{z} = \lambda \boldsymbol{z},$$

$$\mathbf{D}^{-\frac{1}{2}}\mathbf{D}\mathbf{D}^{-\frac{1}{2}} z - \mathbf{D}^{-\frac{1}{2}}\mathbf{W}\mathbf{D}^{-\frac{1}{2}} z = \lambda z$$

$$\boldsymbol{z} = \mathbf{D}^{\frac{1}{2}} \boldsymbol{y}.$$

$$\mathbf{I} z - \mathbf{D}^{-\frac{1}{2}}\mathbf{W}\mathbf{D}^{-\frac{1}{2}} z = \lambda z$$

But solutions to above are solutions to:

$$\mathbf{D}^{-\frac{1}{2}}\mathbf{W}\mathbf{D}^{-\frac{1}{2}} z = \lambda z$$

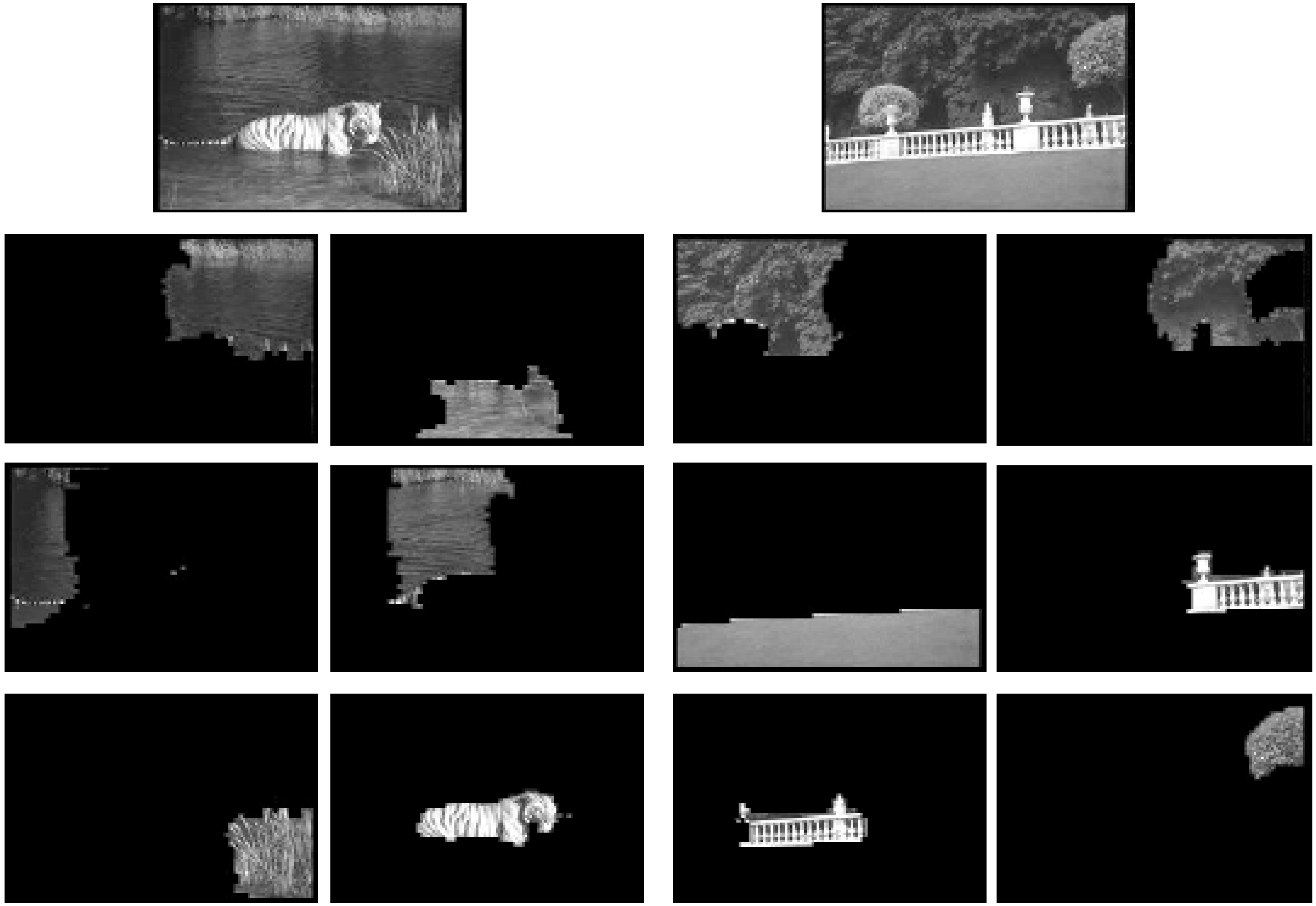
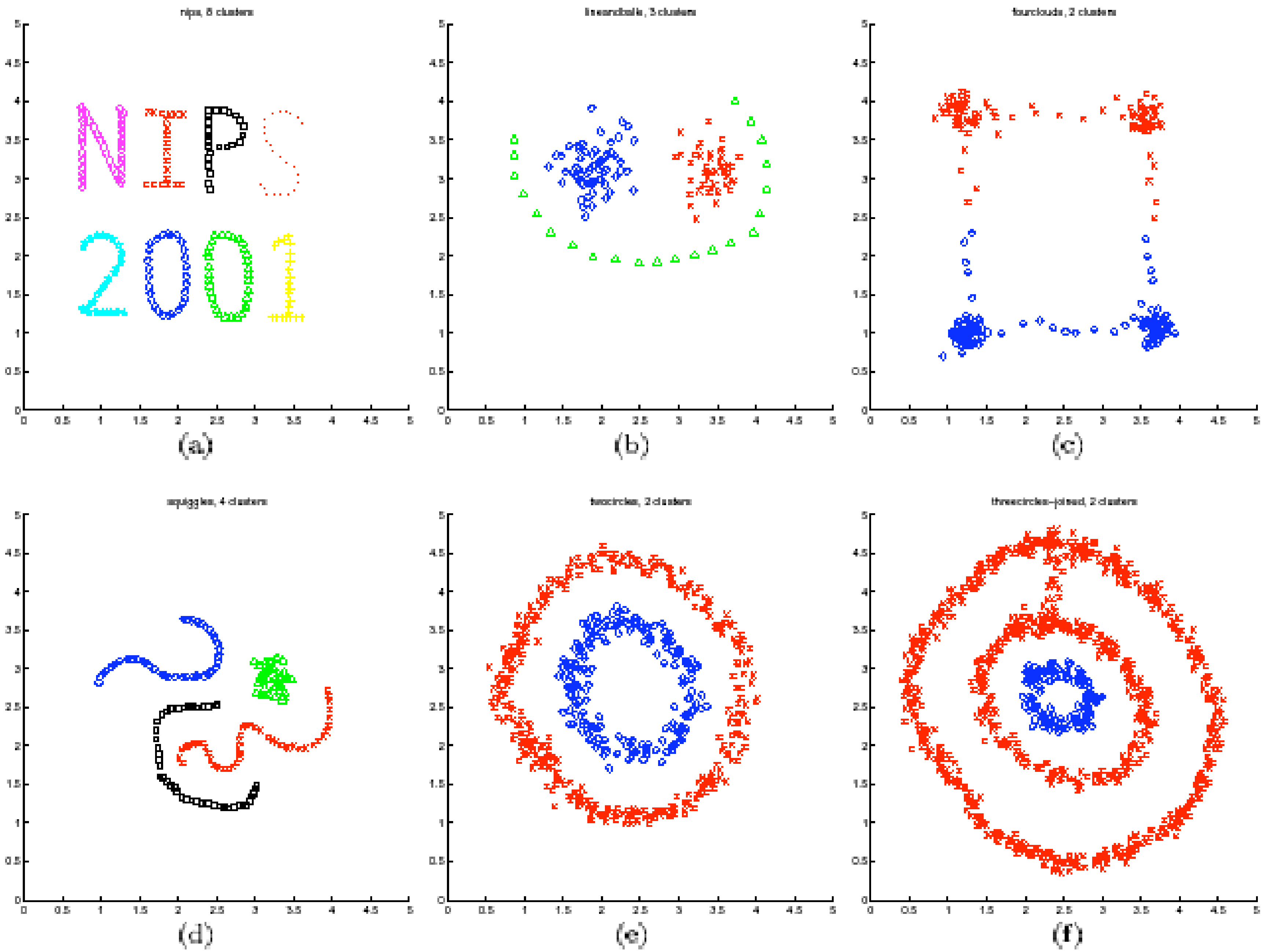


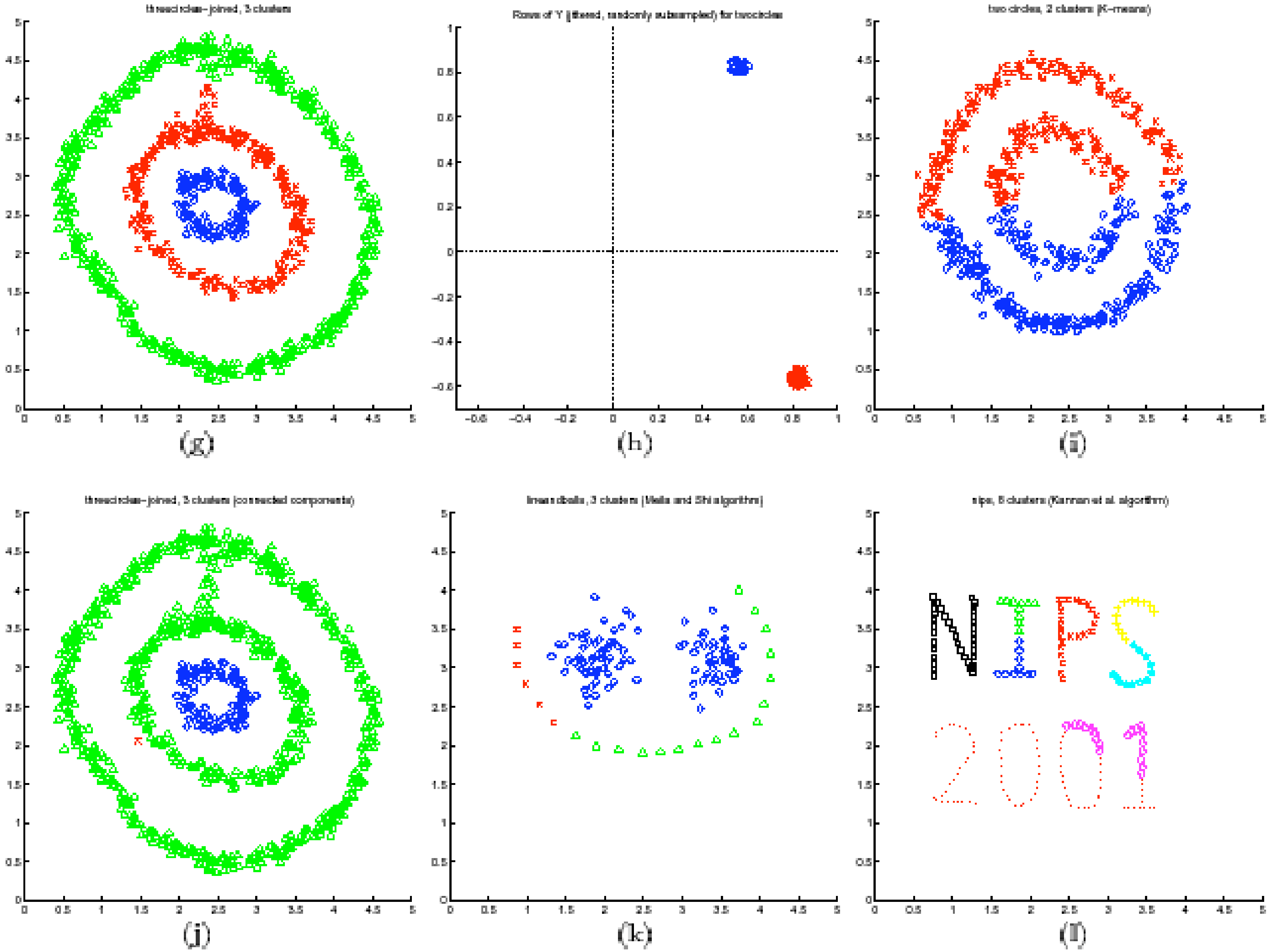
Figure from “Image and video segmentation: the normalised cut framework”, by Shi and Malik, copyright IEEE, 1998

Ng,Jordan,Weiss, 2002

Given a set of points $S = \{s_1, \ldots, s_n\}$ in $\mathbb{R}^l$ that we want to cluster into k subsets:

1. Form the affinity matrix $A \in \mathbb{R}^{n \times n}$ defined by $A_{ij} = \exp(-\|s_i - s_j\|^2 / 2\sigma^2)$ if $i \neq j$, and $A_{ii} = 0$.
2. Define D to be the diagonal matrix whose (i, i)-element is the sum of A's i-th row, and construct the matrix $L = D^{-1/2} A D^{-1/2}$.[1]
3. Find $x_1, x_2, \ldots, x_k$, the k largest eigenvectors of L (chosen to be orthogonal to each other in the case of repeated eigenvalues), and form the matrix $X = [x_1 x_2 \ldots x_k] \in \mathbb{R}^{n \times k}$ by stacking the eigenvectors in columns.
4. Form the matrix Y from X by renormalizing each of X's rows to have unit length (i.e. $Y_{ij} = X_{ij} / (\sum_j X_{ij}^2)^{1/2}$).
5. Treating each row of Y as a point in $\mathbb{R}^k$, cluster them into k clusters via K-means or any other algorithm (that attempts to minimize distortion).
6. Finally, assign the original point s_i to cluster j if and only if row i of the matrix Y was assigned to cluster j.


nips, 8 clusters
lineandballs, 3 clusters
fourclouds, 2 clusters
(a)
(b)
(c)
squiggles, 4 clusters
twocircles, 2 clusters
threecircles-joined, 2 clusters
(d)
(e)
(f)


threecircles-joined, 3 clusters
Rows of Y (jittered, randomly subsampled) for twocircles
two circles, 2 clusters (K-means)
threecircles-joined, 3 clusters (connected components)
lineandballs, 3 clusters (Meila and Shi algorithm)
nips, 8 clusters (Kannan et al. algorithm)
NIPS
2001

(g) (h) (i)

(j) (k) (l)

F igure from “Normalized cuts and image segmentation,” Shi and Malik, copyright IEEE, 2000