Filtering 2

Topics

- Filtering as a linear transform
- Fourier Transform
- Filtering as feature finding
- Filtering for target detection

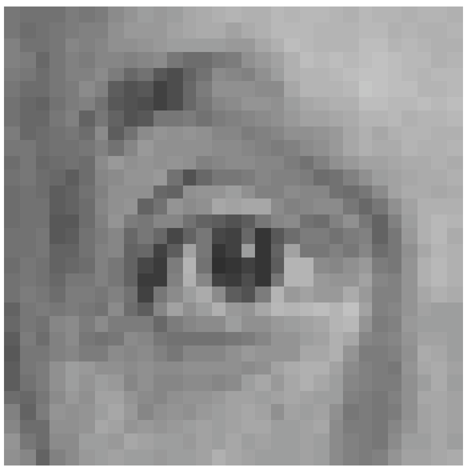
Convolution

$$f[m,n] = I \otimes g = \sum_{k,l} I[m-k, n-l]\, g[k,l]$$

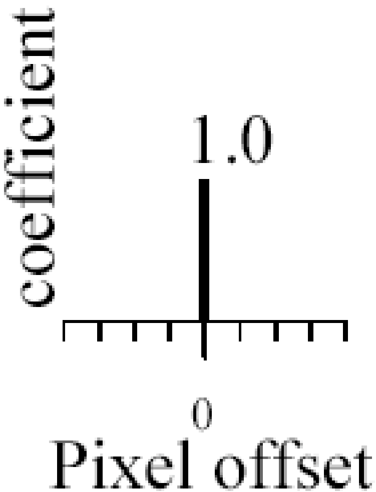
For a filter size N by M,

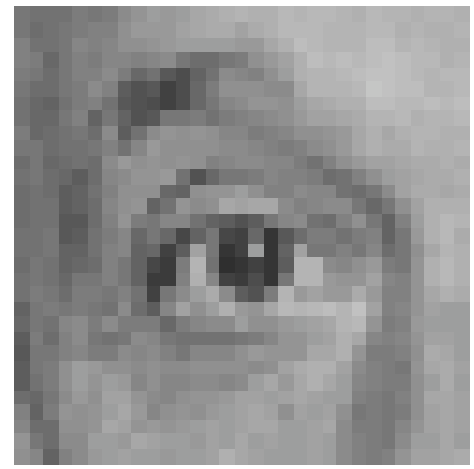
$$R_{ij} = \sum_{u=1:N} \sum_{v=1:M} H_{uv} F_{i-u,\, j-v}$$

Linear filtering (warm-up slide)



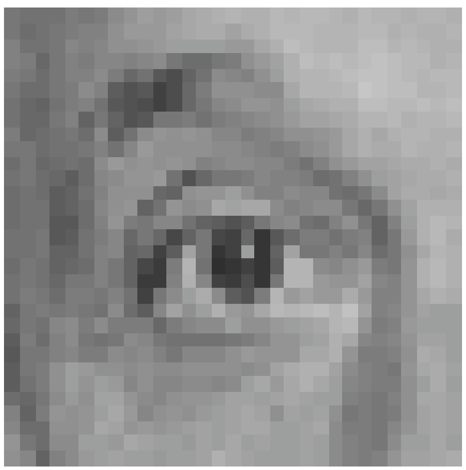
original



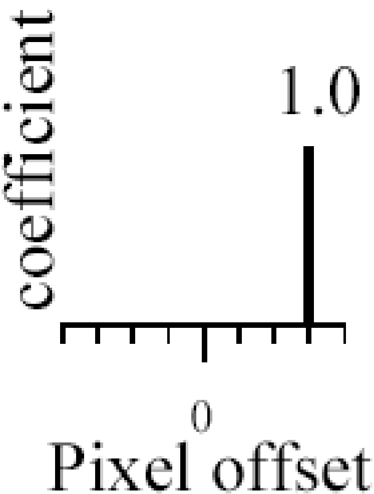


Filtered
(no change)

Linear filtering

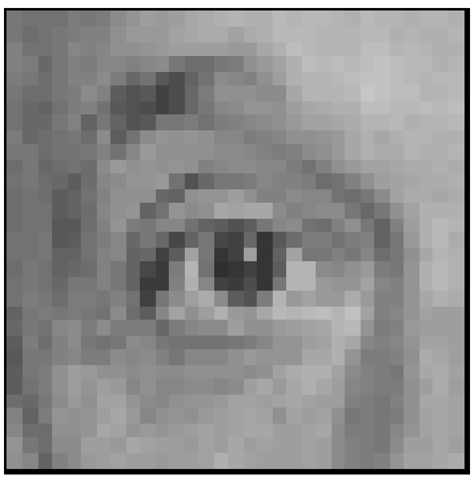


original

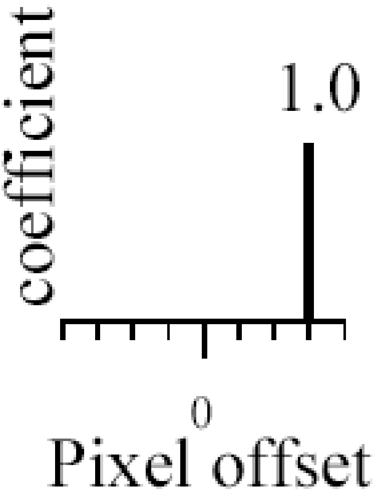


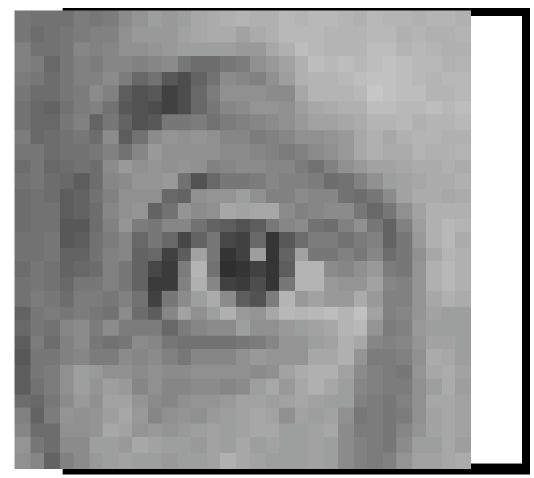
?

shift



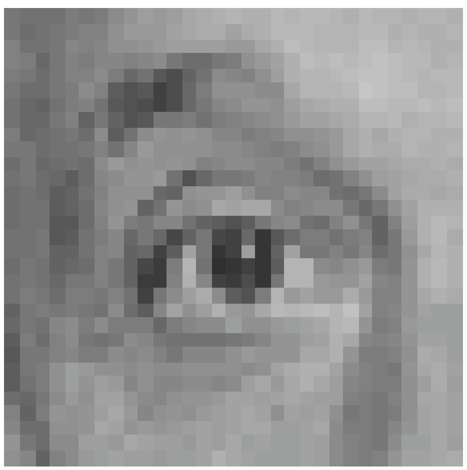
original



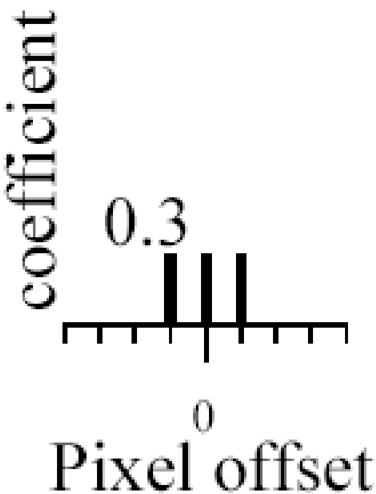


shifted

Linear filtering

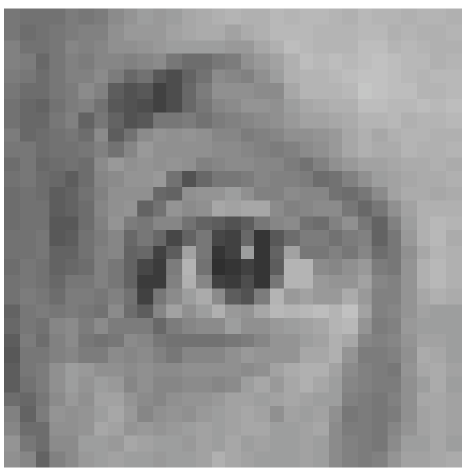


original

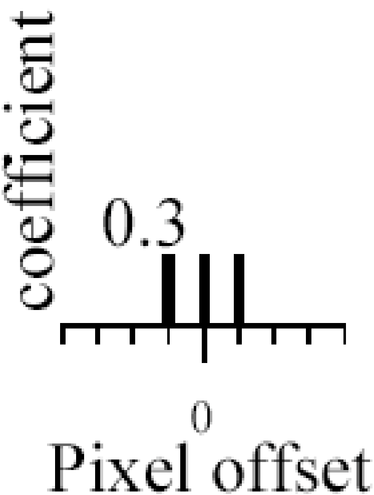


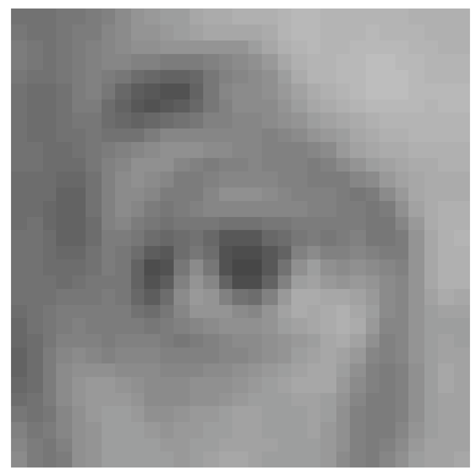
?

Blurring



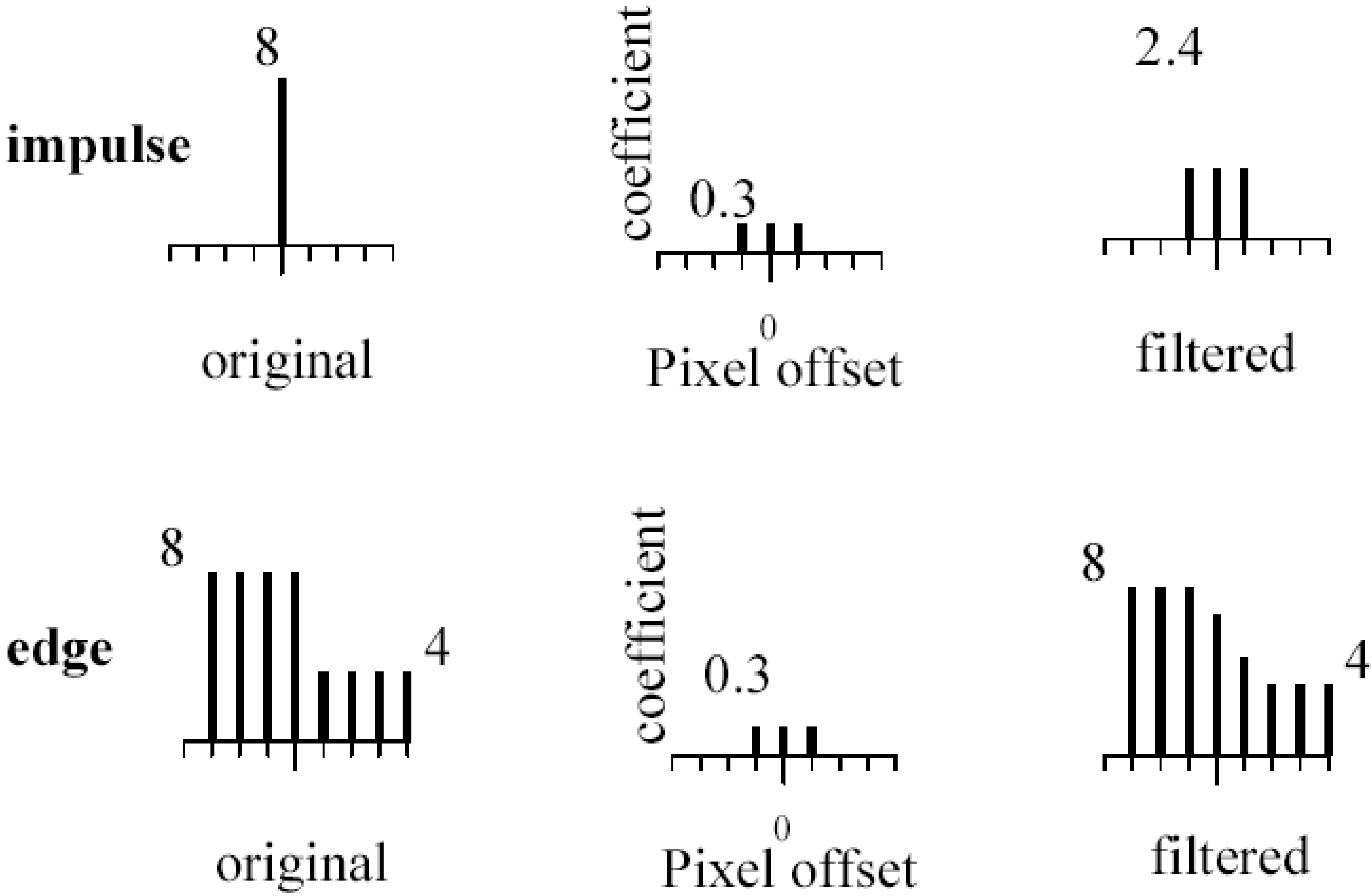
original



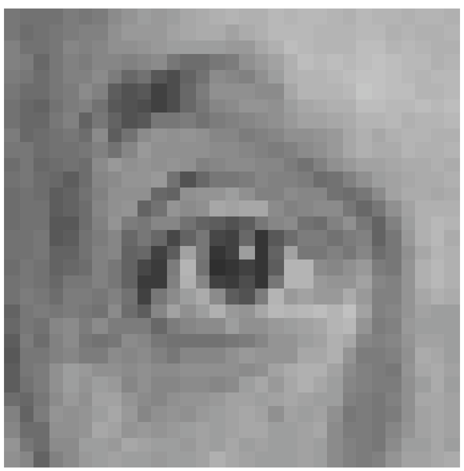


Blurred (filter applied in both dimensions).

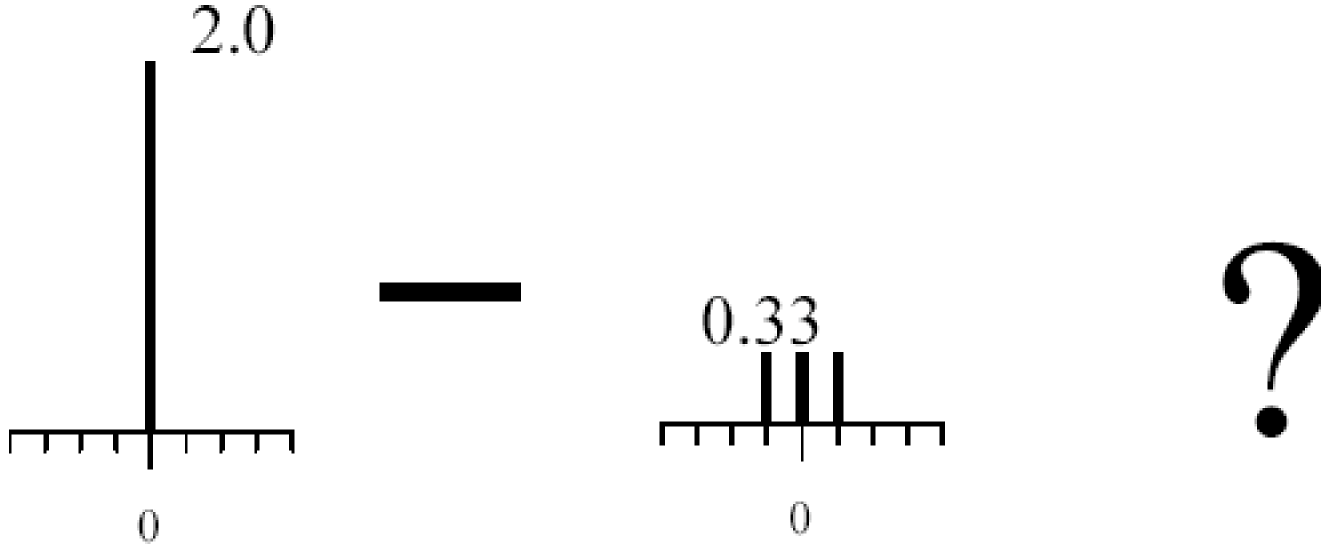
Blur examples



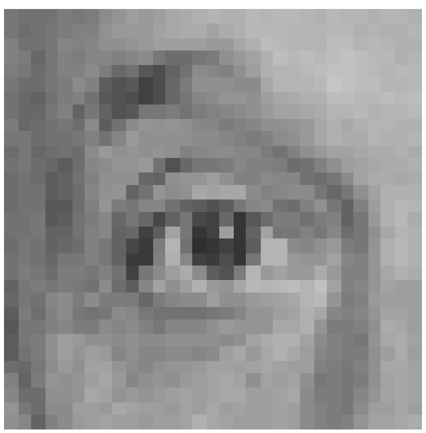
Linear filtering



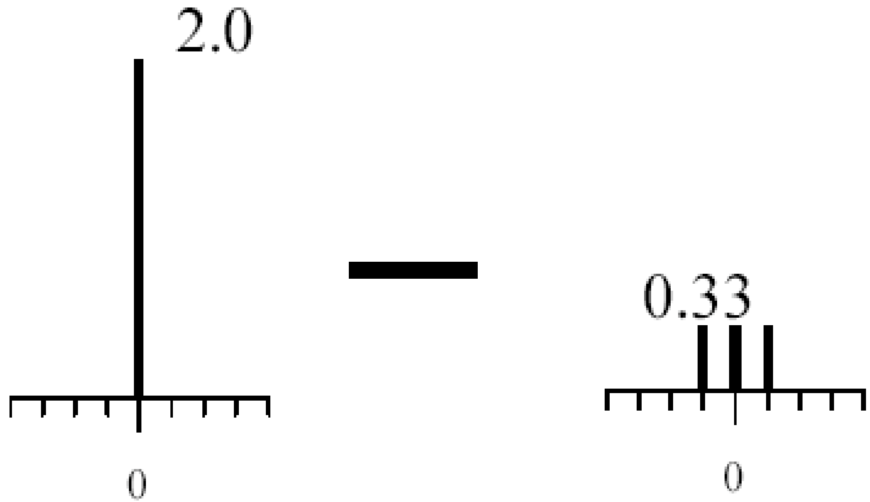
original



Sharpening

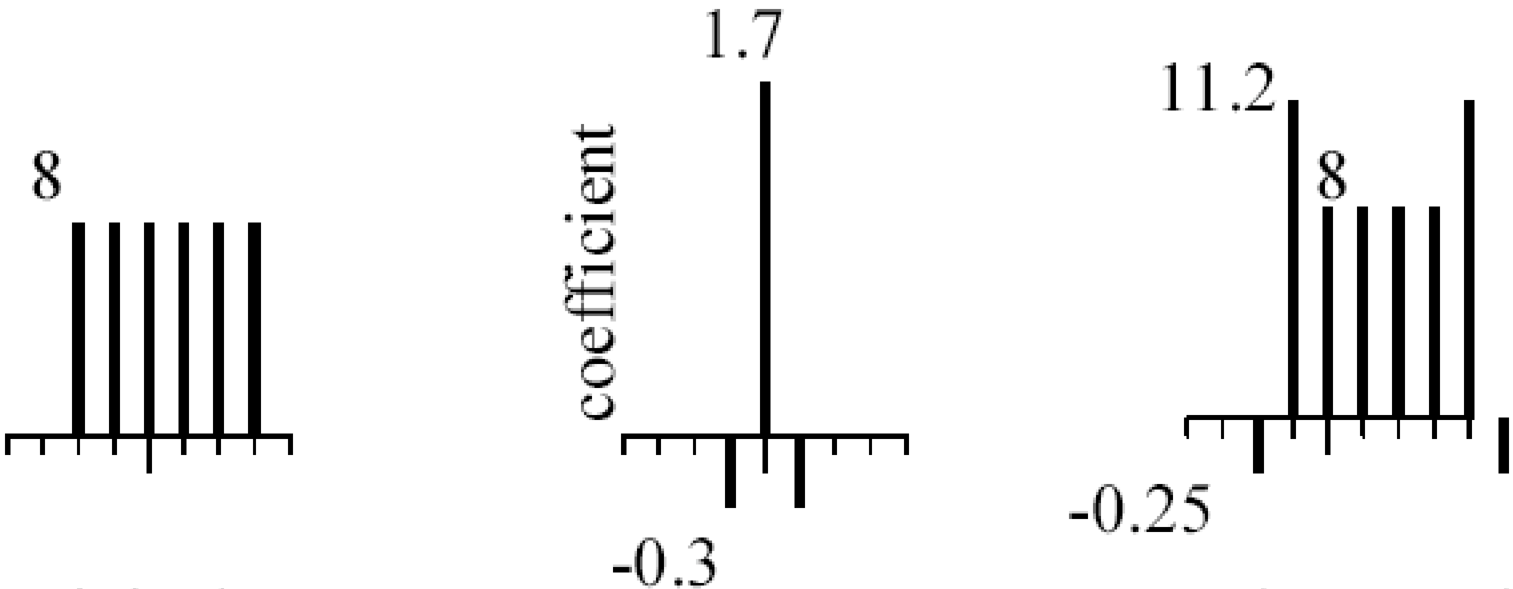


original



Sharpened
original

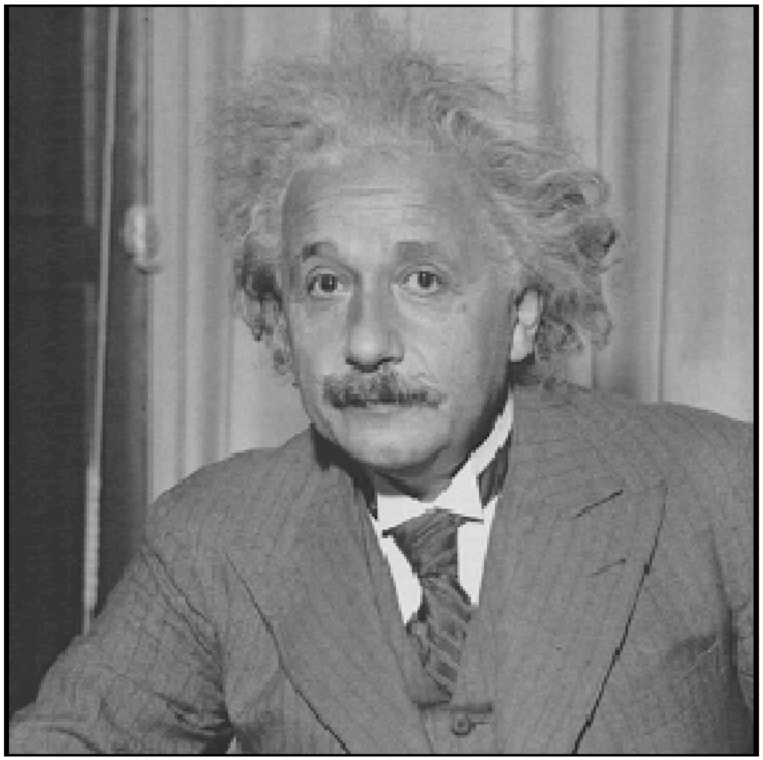
Sharpening example

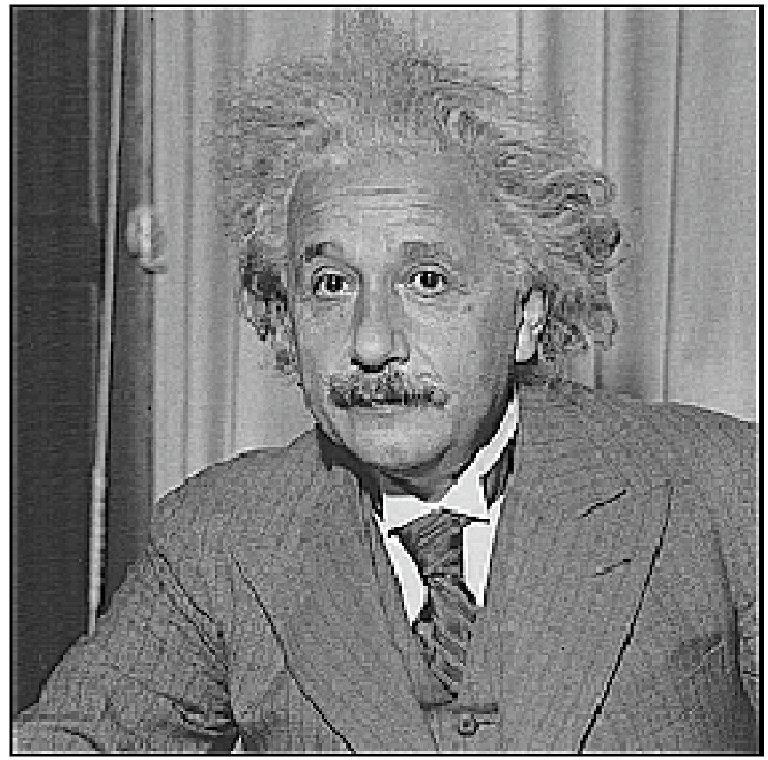


original

Sharpened (differences are accentuated; constant areas are left untouched).

Sharpening





before **after**

Linear image transformations

- In analyzing images, it's often useful to make a change of basis.

transformed image

$$\vec{F} = U\vec{f}$$

Vectorized image

Fourier transform, or
Wavelet transform, or
Steerable pyramid transform

Self-inverting transforms

Same basis functions are used for the inverse transform

$$\vec{f} = U^{-1}\vec{F}$$

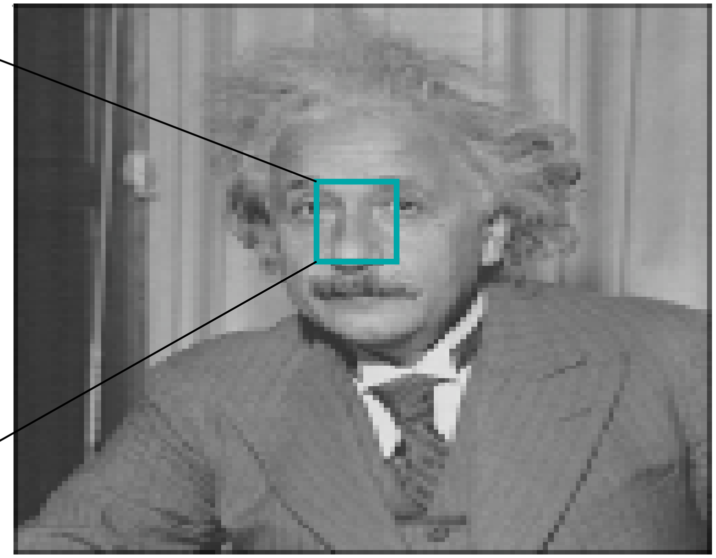
$$= U^{+}\vec{F}$$

U transpose and complex conjugate

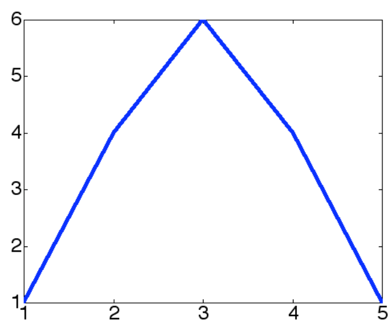
Filtering as a Linear Transform

F =

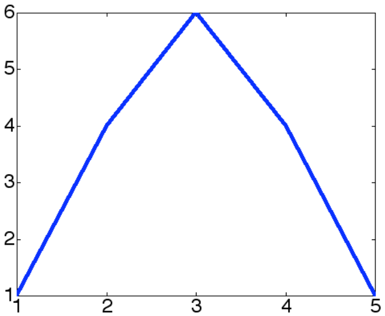
101	101	112	96	124	167	158	134	128	115	114	107	95	149	64	75
117	122	115	114	115	164	156	132	134	128	149	129	139	178	89	52
144	147	139	128	122	162	153	135	131	129	168	152	135	149	154	135
155	163	171	126	124	144	143	134	127	127	162	145	150	140	131	127
133	149	153	111	122	150	140	130	129	138	149	159	149	150	159	158
148	159	133	109	118	152	141	131	127	131	143	166	169	157	149	148
143	144	121	106	118	151	144	130	127	135	134	152	171	173	167	163
141	147	115	105	129	149	143	133	128	136	140	145	154	171	177	179
145	141	118	115	142	151	150	143	131	137	143	139	148	157	168	175
145	137	122	134	155	163	153	147	132	124	132	147	149	154	166	166
152	141	128	145	165	171	166	152	138	124	128	147	164	159	165	164
153	159	130	157	175	183	182	161	144	129	139	135	159	169	176	167
161	147	127	156	178	192	183	160	143	137	152	129	158	158	178	178
157	142	114	129	161	168	163	141	120	139	148	125	168	172	162	179
157	152	70	87	129	139	129	103	67	63	100	98	175	174	166	167
157	155	116	68	71	83	77	56	94	136	142	169	177	172	176	163

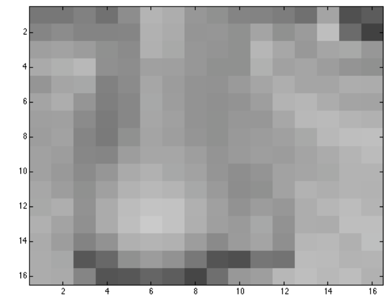


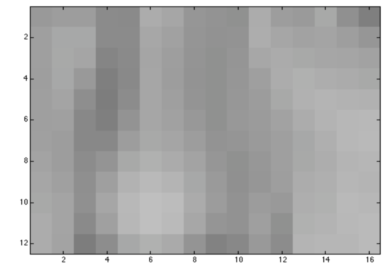
h = 1
4
6
4
1



F convolved with H = ?



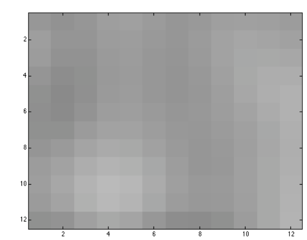




F’ = H*F

y filtering

$$
H = \begin{matrix}
1&4&6&4&1&0&0&0&0&0&0&0&0&0&0&0\\
0&1&4&6&4&1&0&0&0&0&0&0&0&0&0&0\\
0&0&1&4&6&4&1&0&0&0&0&0&0&0&0&0\\
0&0&0&1&4&6&4&1&0&0&0&0&0&0&0&0\\
0&0&0&0&1&4&6&4&1&0&0&0&0&0&0&0\\
0&0&0&0&0&1&4&6&4&1&0&0&0&0&0&0\\
0&0&0&0&0&0&1&4&6&4&1&0&0&0&0&0\\
0&0&0&0&0&0&0&1&4&6&4&1&0&0&0&0\\
0&0&0&0&0&0&0&0&1&4&6&4&1&0&0&0\\
0&0&0&0&0&0&0&0&0&1&4&6&4&1&0&0\\
0&0&0&0&0&0&0&0&0&0&1&4&6&4&1&0\\
0&0&0&0&0&0&0&0&0&0&0&1&4&6&4&1
\end{matrix}
$$



H*F*H'

x & y filtering

An example of such a transform: the Fourier transform

discrete domain

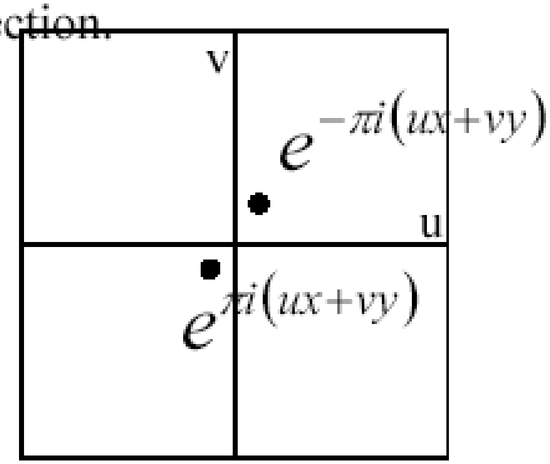
Forward transform

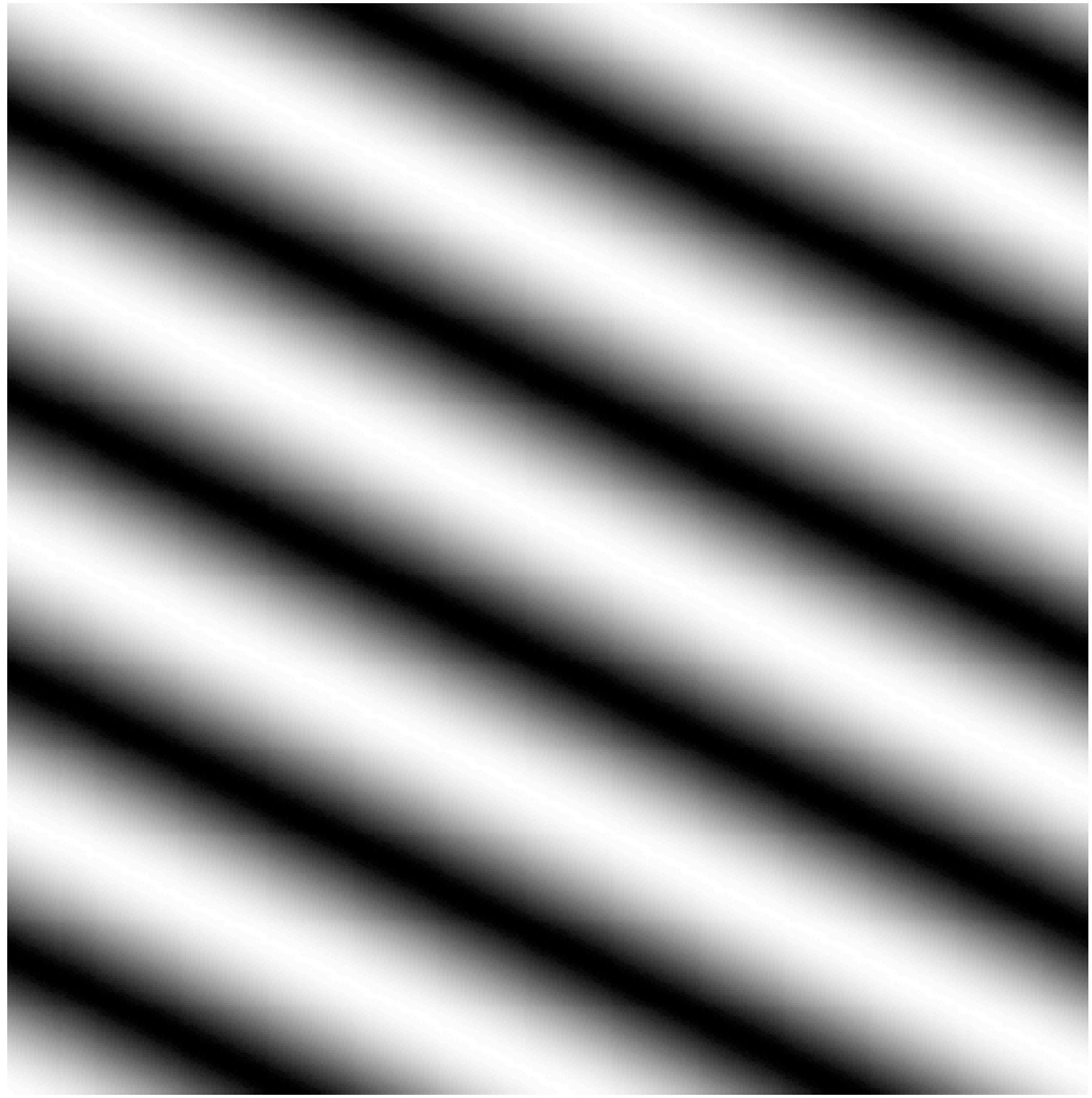
$$F[m,n]=\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} f[k,l]e^{-\pi i\left(\frac{km}{M}+\frac{\mathrm{ln}}{N}\right)}$$

Inverse transform

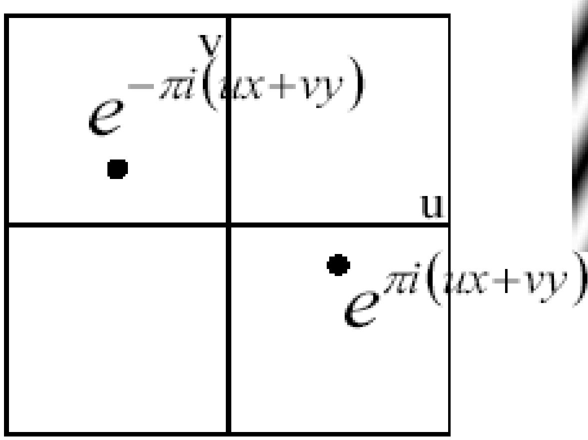
$$f[k,l]=\frac{1}{MN}\sum_{k=0}^{M-1}\sum_{l=0}^{N-1} F[m,n]e^{+\pi i\left(\frac{km}{M}+\frac{\mathrm{ln}}{N}\right)}$$

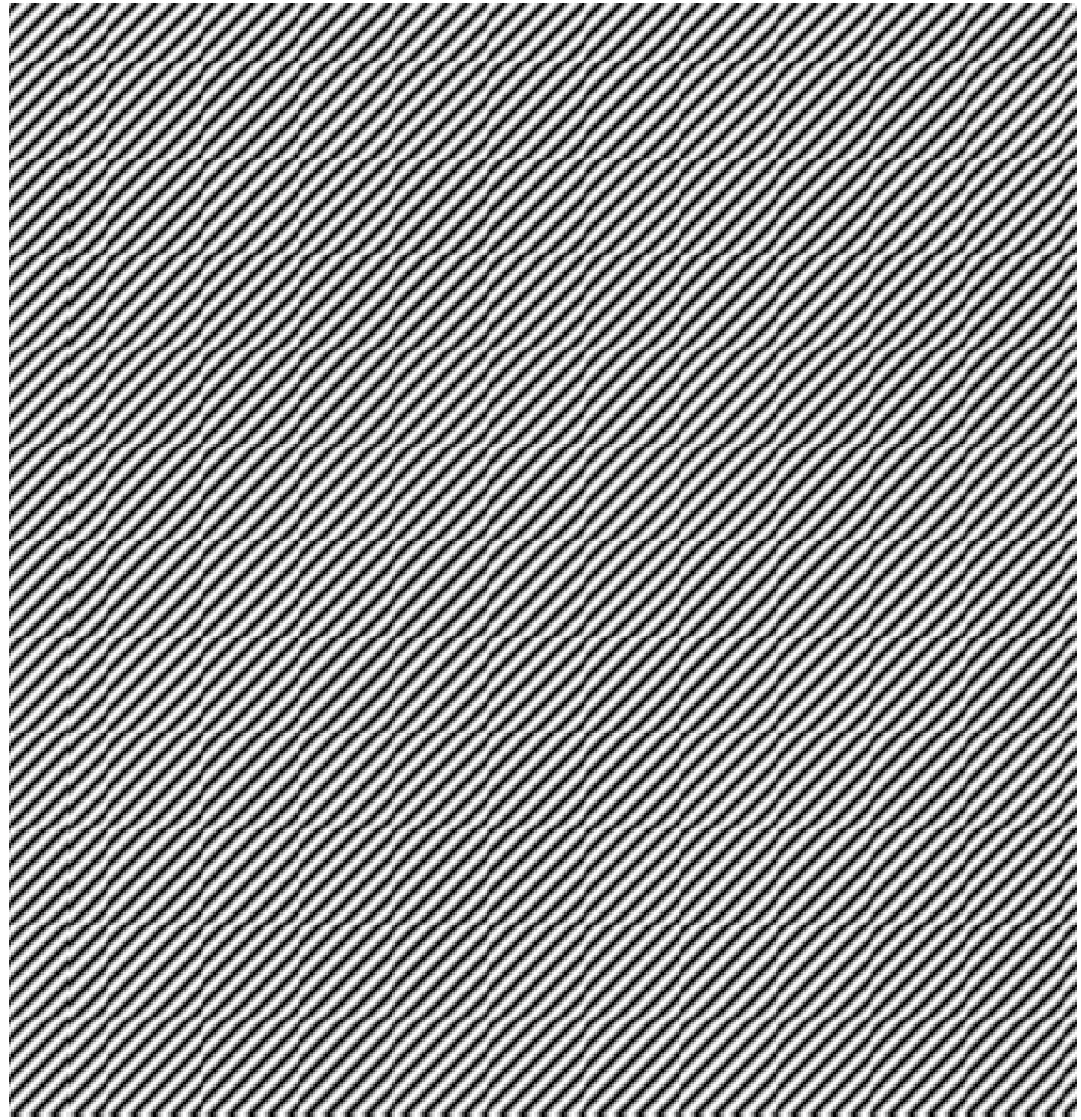
To get some sense of what basis elements look like, we plot a basis element --- or rather, its real part --- as a function of x,y for some fixed u, v. We get a function that is constant when (ux+vy) is constant. The magnitude of the vector (u, v) gives a frequency, and its direction gives an orientation. The function is a sinusoid with this frequency along the direction, and constant perpendicular to the direction.





Here u and v are larger than in the previous slide.





Phase and Magnitude

- Fourier transform of a real function is complex
  - difficult to plot, visualize
  - instead, we can think of the phase and magnitude of the transform
- Phase is the phase of the complex transform
- Magnitude is the magnitude of the complex transform
- Curious fact
  - all natural images have about the same magnitude transform
  - hence, phase seems to matter, but magnitude largely doesn't
- Demonstration
  - Take two pictures, swap the phase transforms, compute the inverse - what does the result look like?

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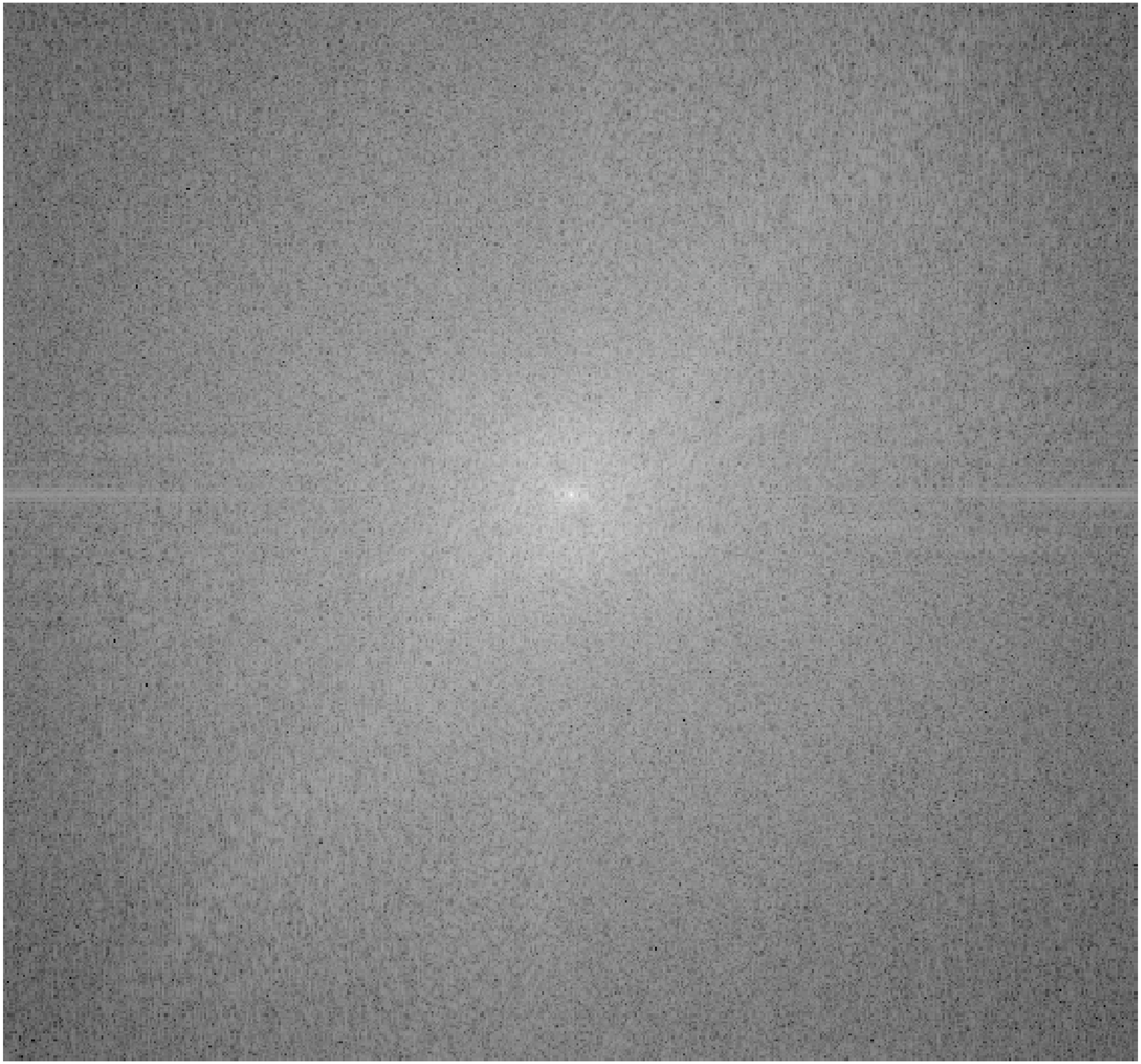
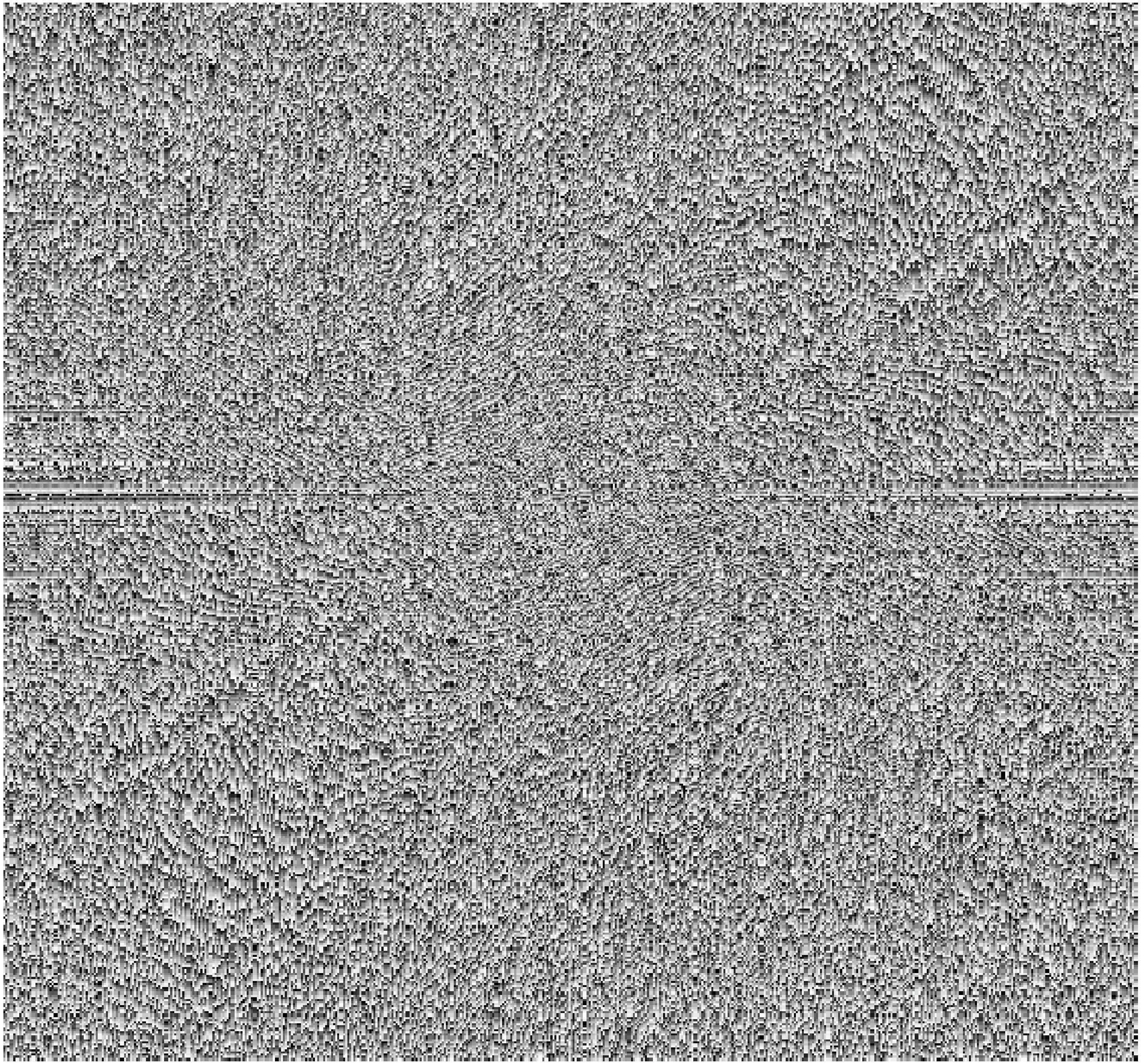
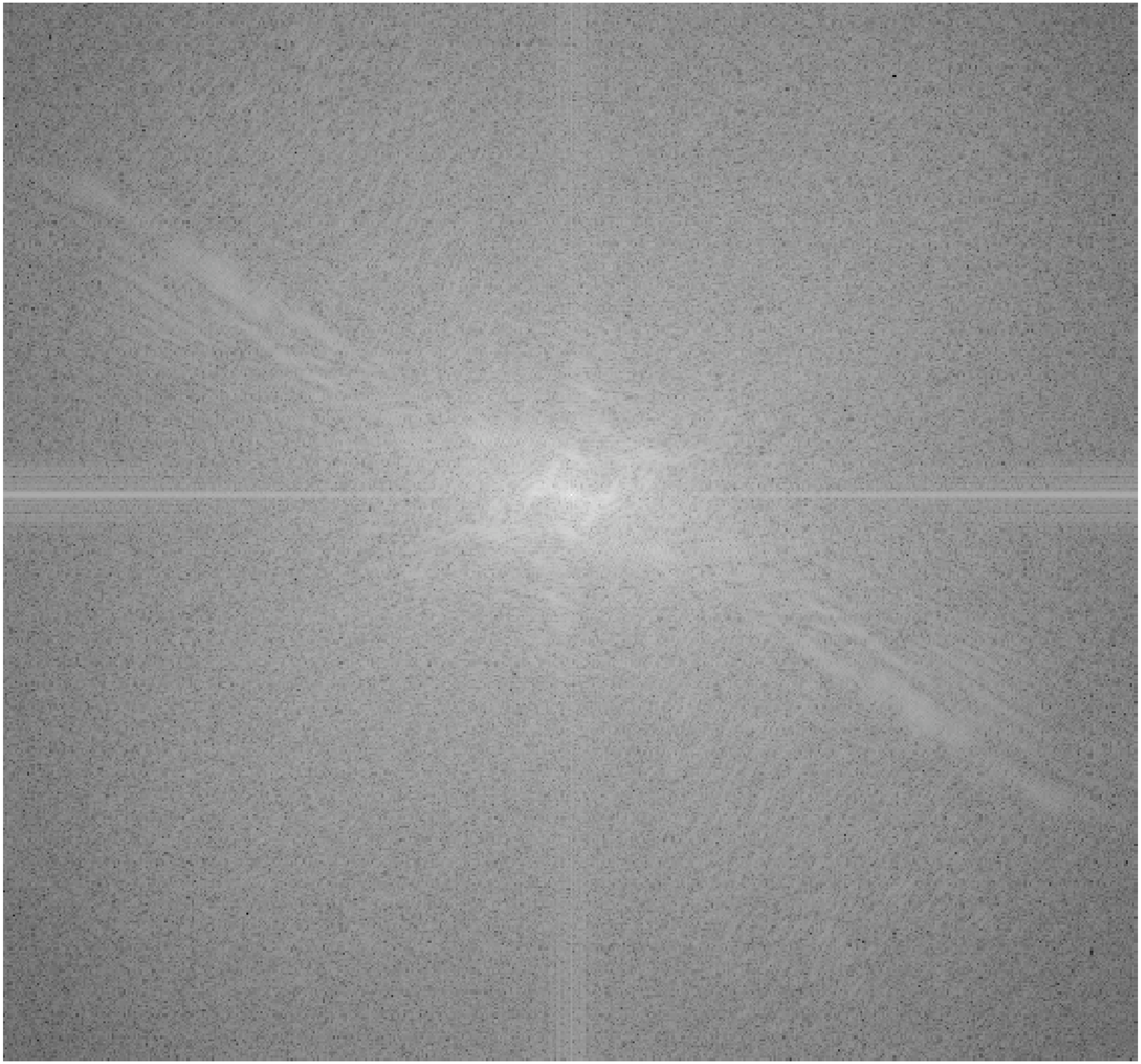
This is the
magnitude
transform
of the
cheetah pic



This is the phase transform of the cheetah pic

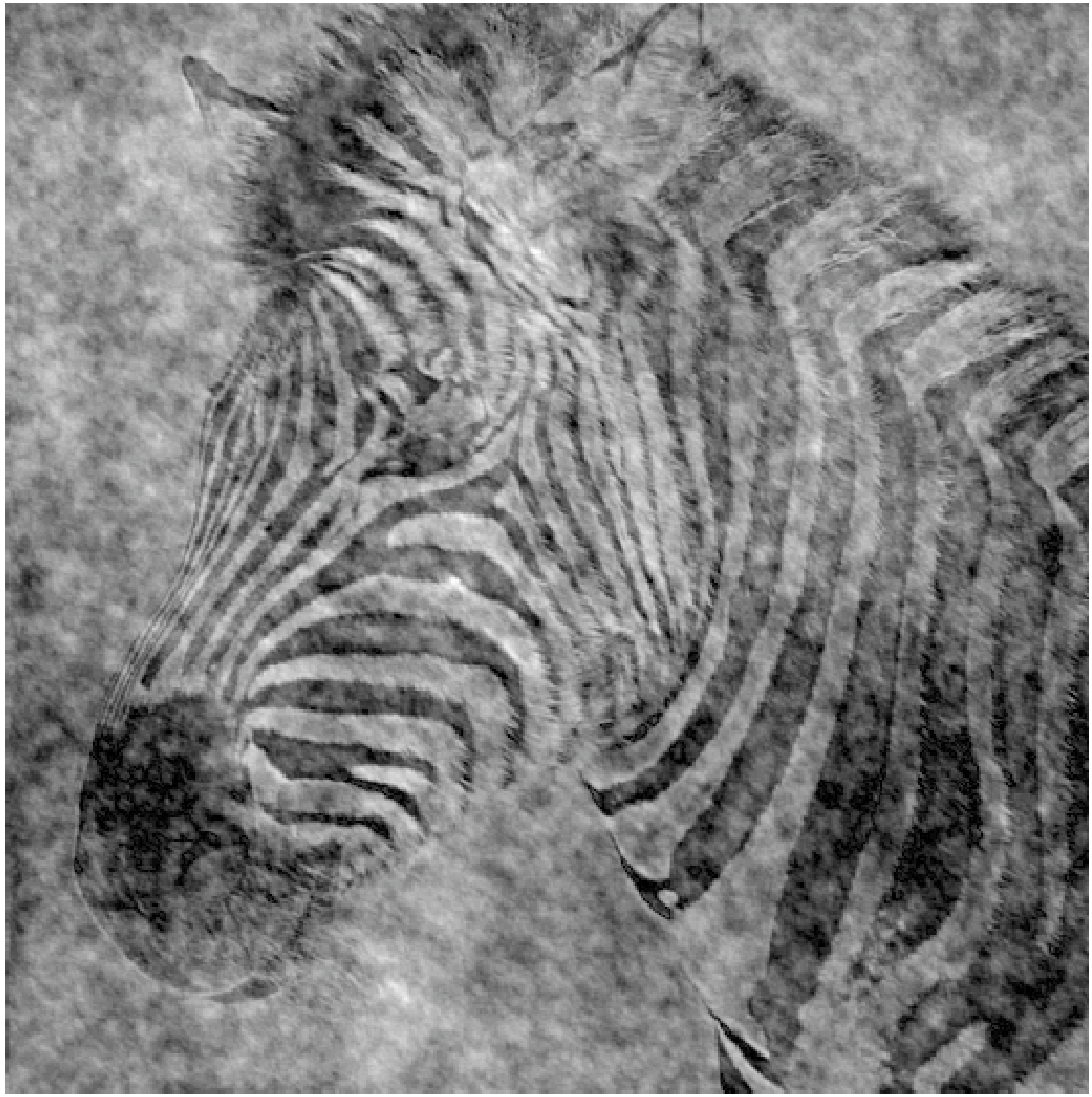


This is the magnitude transform of the zebra pic



This is the phase transform of the zebra pic

Reconstruction with zebra phase, cheetah magnitude



Reconstruction with cheetah phase, zebra magnitude

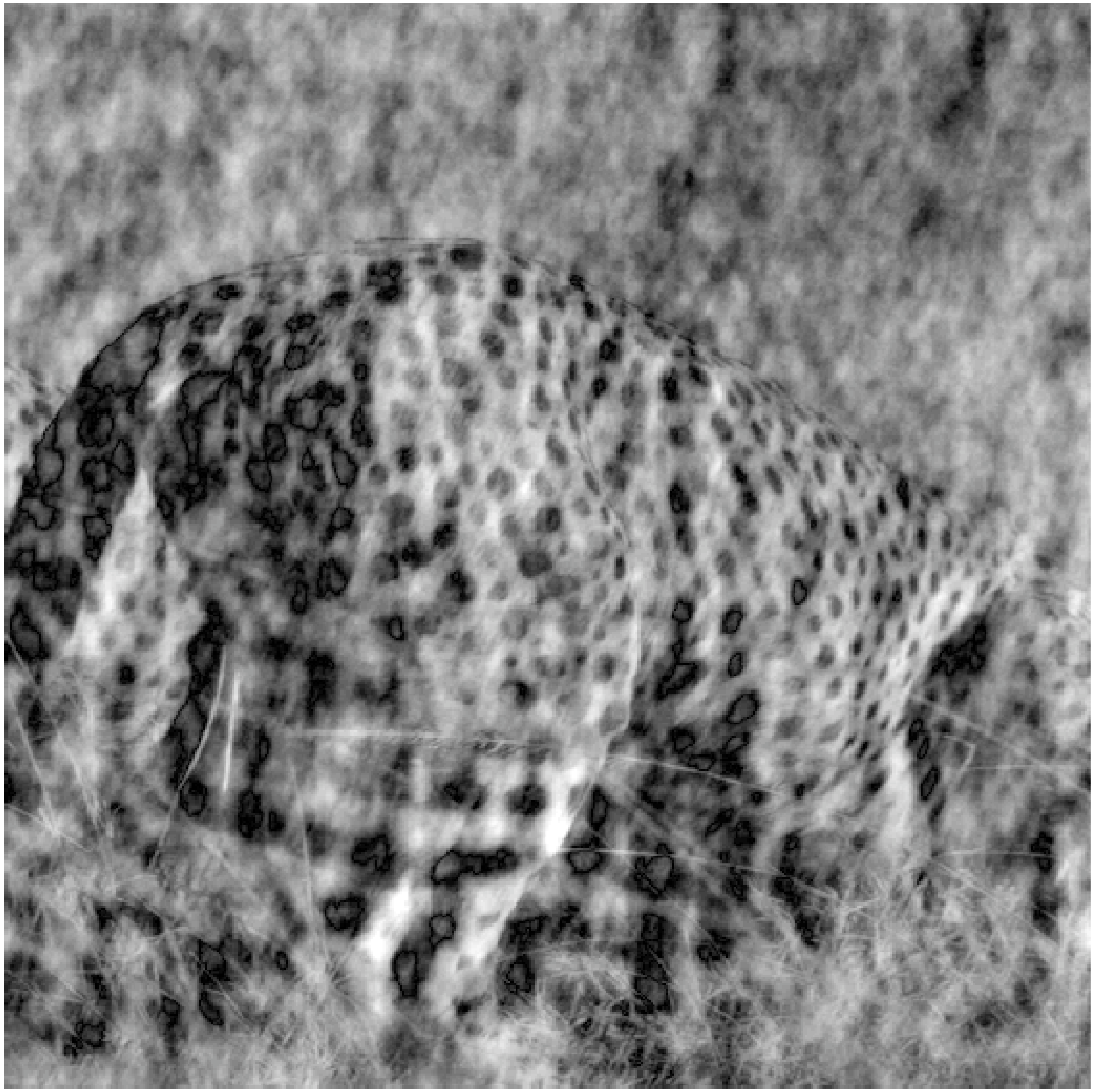
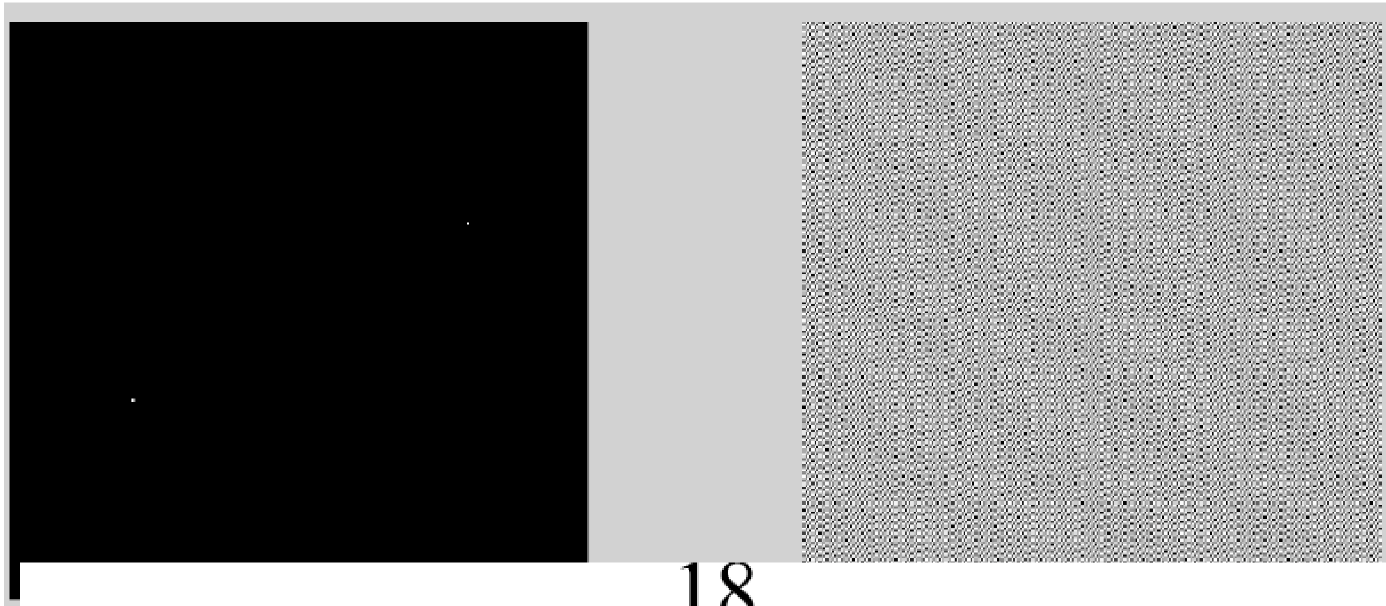
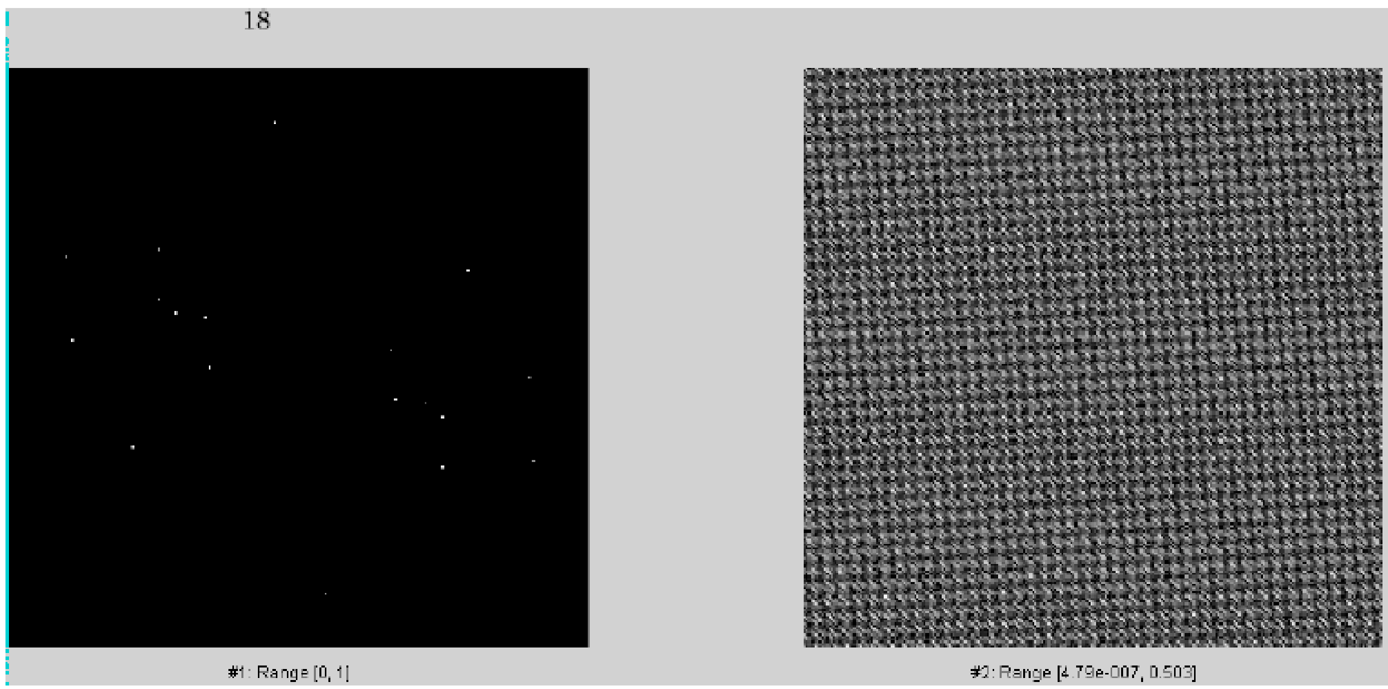


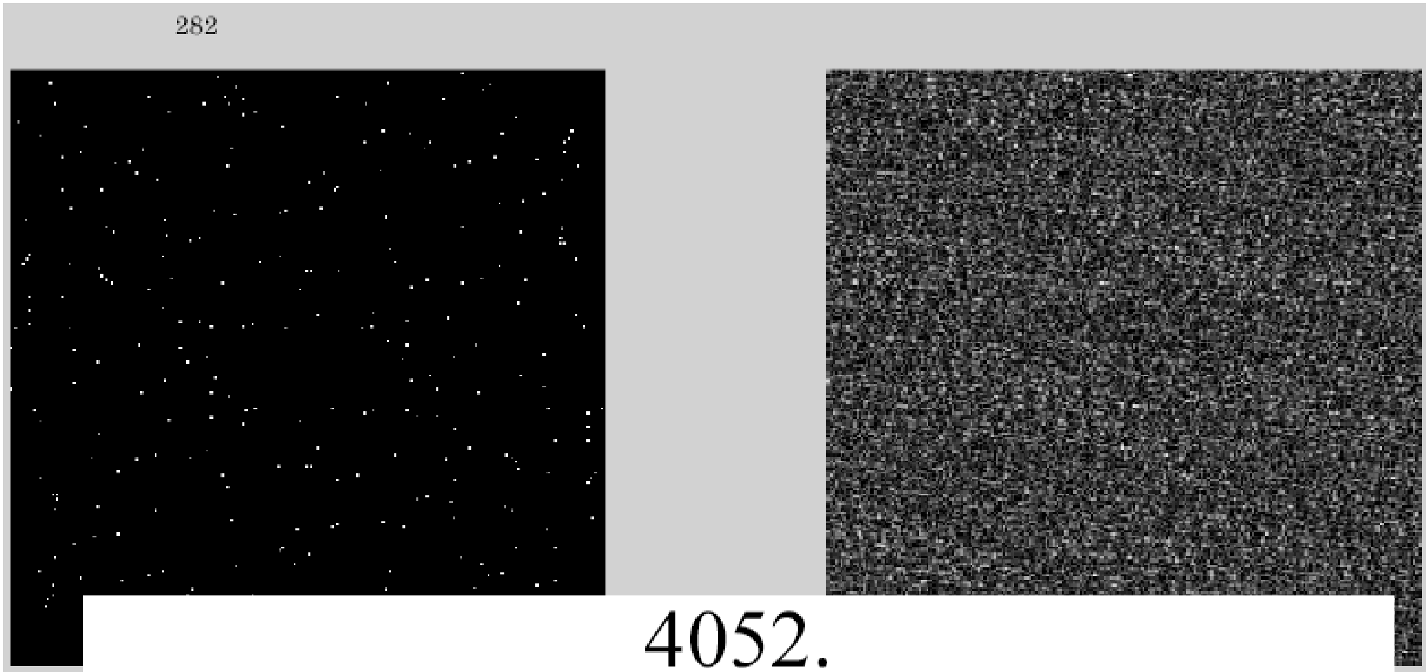
Image Synthesis with Fourier Basis



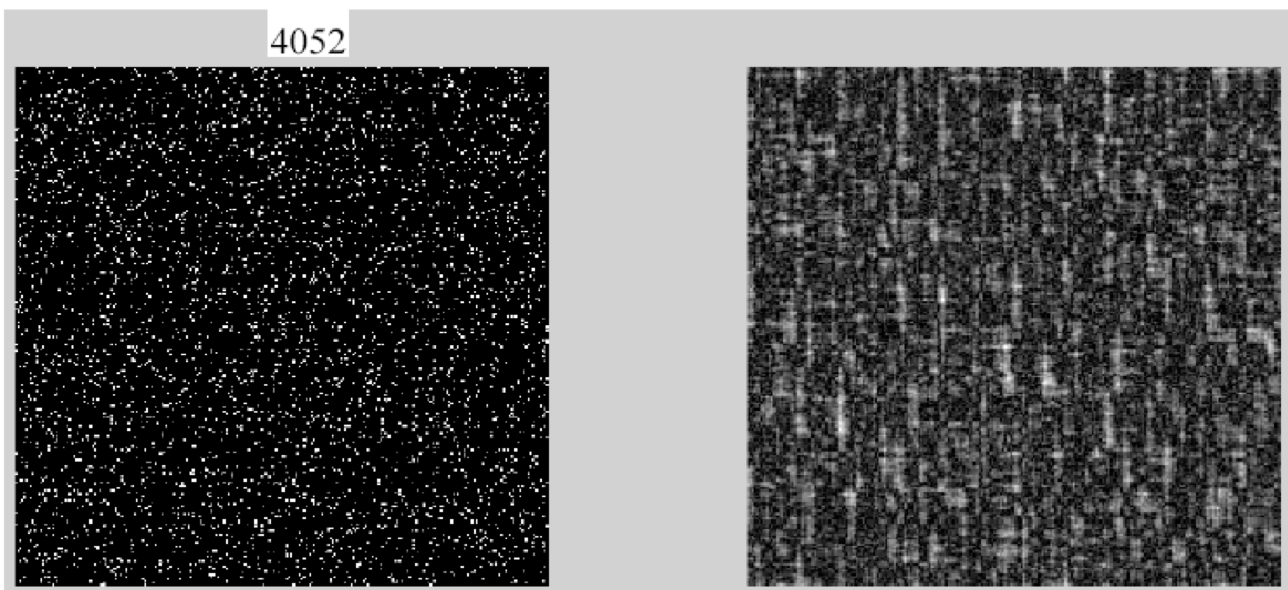
18



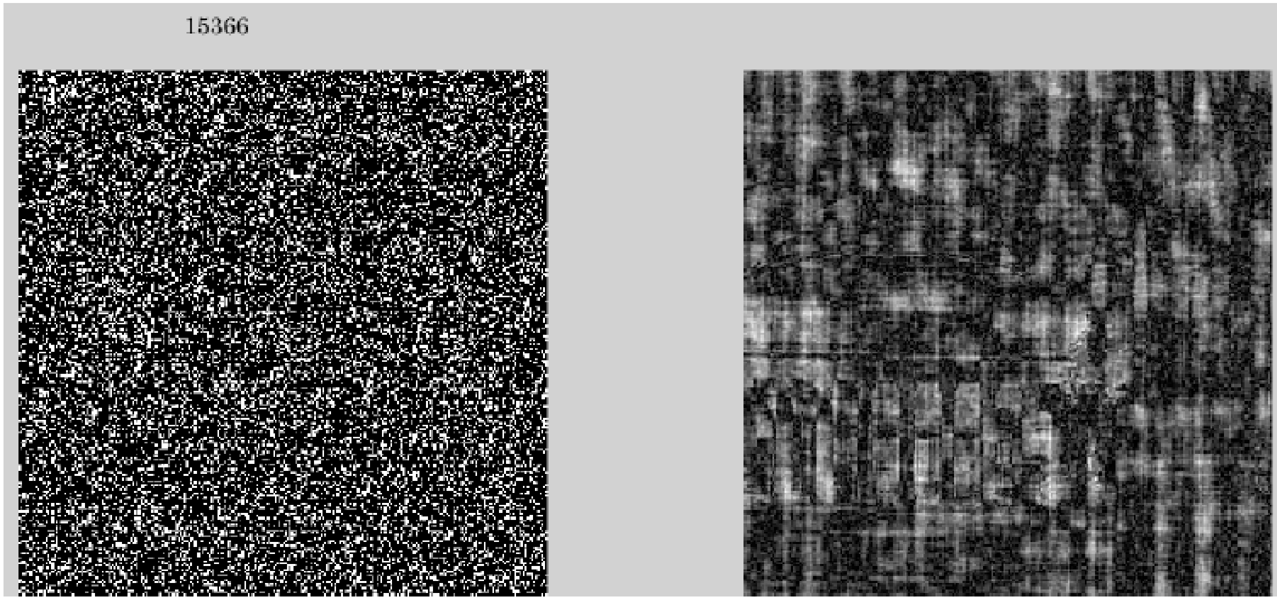
282



4052.



15366



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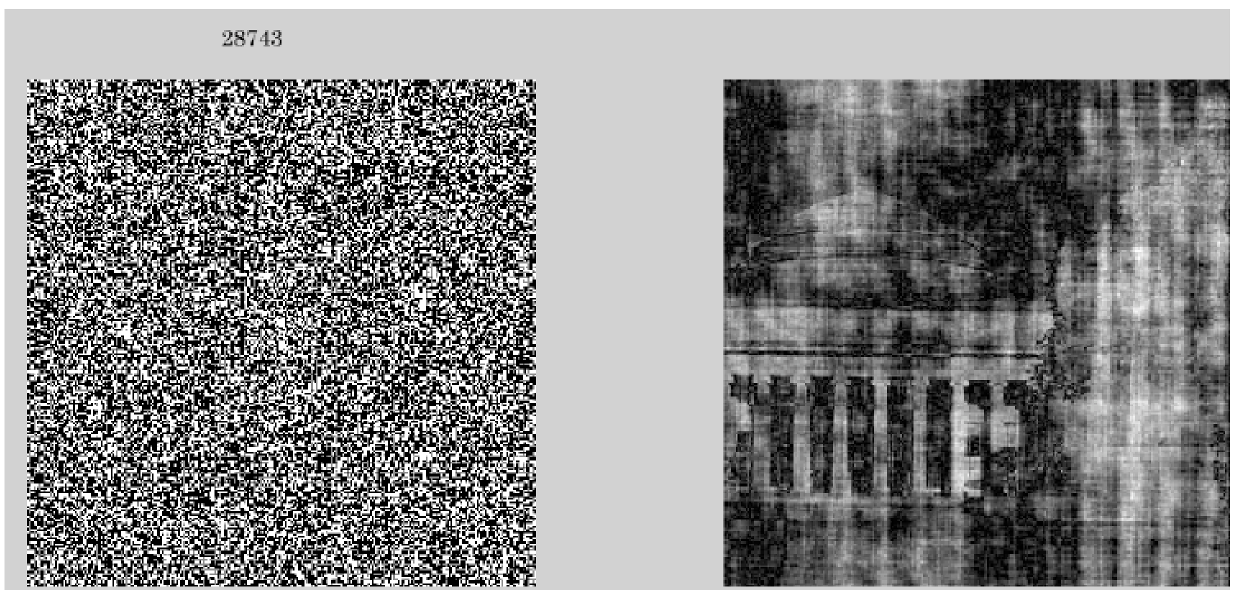

65536.

TABLE 7.1 A variety of functions of two dimensions and their Fourier transforms. This table can be used in two directions (with appropriate substitutions for u, v and x, y) because the Fourier transform of the Fourier transform of a function is the function. Observant readers may suspect that the results on infinite sums of δ functions contradict the linearity of Fourier transforms. By careful inspection of limits, it is possible to show that they do not (see, e.g., Bracewell, 1995). Observant readers may also have noted that an expression for $\mathcal{F}(\frac{\partial f}{\partial y})$ can be obtained by combining two lines of this table.

Function	Fourier transform
$g(x, y)$	$\iint_{-\infty}^{\infty} g(x, y)e^{-i2\pi(ux+vy)}\,dx\,dy$
$\iint_{-\infty}^{\infty} \mathcal{F}(g(x, y))(u, v)e^{i2\pi(ux+vy)}\,du\,dv$	$\mathcal{F}(g(x, y))(u, v)$
$\delta(x, y)$	1
$\frac{\partial f}{\partial x}(x, y)$	$u\mathcal{F}(f)(u, v)$
$0.5\delta(x + a, y) + 0.5\delta(x - a, y)$	$\cos 2\pi au$
$e^{-\pi(x^2+y^2)}$	$e^{-\pi(u^2+v^2)}$
$box_1(x, y)$	$\frac{\sin u}{u}\frac{\sin v}{v}$
$f(ax, by)$	$\frac{\mathcal{F}(f)(u/a, v/b)}{ab}$
$\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\delta(x - i, y - j)$	$\sum_{i=-\infty}^{\infty}\sum_{j=-\infty}^{\infty}\delta(u - i, v - j)$
$(f**g)(x, y)$	$\mathcal{F}(f)\mathcal{F}(g)(u, v)$
$f(x - a, y - b)$	$e^{-i2\pi(au+bv)}\mathcal{F}(f)$
$f(x\cos\theta - y\sin\theta, x\sin\theta + y\cos\theta)$	$\mathcal{F}(f)(u\cos\theta - v\sin\theta, u\sin\theta + v\cos\theta)$