

Load up the images and produce point matches. Use the zoom feature to produce better matches. Type `help collect_point_matches` for more info.

```
im1 = imread('shed1.jpg');
im2 = imread('shed2.jpg');
[x1, x2, tri_pts] = collect_point_matches(im1, im2)
```

Now make `x1` & `x2` into homogeneous vectors by adding columns of ones.

### 1 ) Implementing the Normalized 8-Point Algorithm

Now we have a set of point correspondences we are ready to calculate the fundamental matrix. As mentioned in Forsyth & Ponce (pg. 220), computing the fundamental matrix directly behaves rather poorly. Hartley showed this was primarily due to poor scaling, and rescaling improves things substantially. Rescaling involves:

Normalize each set of image points (left and right separately) so their mean is zero and their root mean squared distance from the origin is  $\sqrt{2}$ .

$$T_{norm} = \begin{bmatrix} \frac{\sqrt{2}}{R} & 0 & \frac{\sqrt{2}}{R} \bar{x} \\ 0 & \frac{\sqrt{2}}{R} & \frac{\sqrt{2}}{R} \bar{y} \\ 0 & 0 & 1 \end{bmatrix}$$

$$\text{where } R = \sqrt{\frac{1}{N} \sum_{i=1}^N \left( (x_i - \bar{x})^2 + (y_i - \bar{y})^2 \right)}$$

$$\text{and } \bar{x} = \frac{1}{N} \sum_{i=1}^N x_i \quad \bar{y} = \frac{1}{N} \sum_{i=1}^N y_i$$

Write a function determines the homogeneous transformation to perform this normalization on any set of 2D homogeneous points, so that

```
% x_norm = T_norm*X
%
% defines an x_norm with a mean of 0 and rms
% (root mean squared) distance from the origin
% of sqrt(2).
```

Normalize **x1** and **x2**.

**Now estimate the fundamental matrix F from normalized points.**

We want to find the fundamental matrix **F** that relates the location of a point **x1** in image 1 with its location **x2** in image 2.

Writing

$$x_2^T F x_1 = 0 \quad F = \begin{bmatrix} f_{11} & f_{12} & f_{13} \\ f_{21} & f_{22} & f_{23} \\ f_{31} & f_{32} & f_{33} \end{bmatrix}$$

Rewrite as a dot product for each point:

$$\vec{q} \cdot \vec{f} = \begin{bmatrix} u_2 u_1 & u_2 v_1 & u_2 & v_2 u_1 & v_2 v_1 & v_2 & u_1 & v_1 & 1 \end{bmatrix} \begin{bmatrix} f_{11} \\ f_{12} \\ f_{13} \\ f_{21} \\ f_{22} \\ f_{23} \\ f_{31} \\ f_{32} \\ f_{33} \end{bmatrix}$$

Form the above left-hand product  $q$  for each of the  $N$  matches and stack them into one large matrix  $A$ . Solve for  $f$  using  $[U, S, V] = \text{svd}(A)$ , where  $f$  is the last column in  $V$ .

Reshape  $f$  into  $F$ .

Enforce Singularity

The fundamental matrix is a  $3 \times 3$  matrix of rank 2, therefore it has singular eigenvalues. It is unlikely your matrix obeys this. Enforce singularity by

$[U, S, V] = \text{svd}(F)$ ;

$S(3,3) = 0$ ;

$F_n = U * S * V'$

Because we normalized the points, this  $F_n$  is not the right  $F_n$ . The correct  $F_n$  is given by:

$$F_n = T\_norm\_2' * F_n * T\_norm\_1$$

If things went well, points in one image will specify an epipolar line in the other. View them using: `epipolar_viewer(Fn, im1, im2)` and check how good your points are.

## 2 ) Stereo reconstruction

These cameras are calibrated. The intrinsic parameter matrices are in the file `C1_C2.mat`.

Use these to compute the Essential matrix via:

$$C_2^t f C_1 = E$$

Recall  $E$  has the extrinsic parameters between the two cameras, and these can be extracted from:

$$E = USV'$$

e.g. the svd of  $E$ , and the rotation matrix  $R$  and translation vectors are given by:

$$R = UWV' \quad \text{or} \quad R = UW'V' \quad \vec{t} = u_3 \quad \text{or} \quad \vec{t} = -u_3$$

$$W = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

which results in a four way ambiguity. Use these results to compute  $R$  and  $t$ .

Then the projection matrices can be written:

$$M1 = C1 * [\text{eye}(3) \ 0 \ 0 \ 0]'$$

$$M2 = C2 * [R \ t] \text{ (with four possibilities)}$$

Now for each point, solve for its  $x, y, z$  coordinates using the method outlined in class.

You will get a 4-D homogeneous vector. Divide all the coordinates by the 4<sup>th</sup> to get the  $x, y, z$  coordinates.

Reproject the points to see which of the four possibilities is correct. In addition, the reprojection will tell you something about how good your reconstruction is.