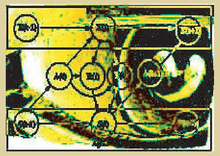

Decision-making with Monetary Value Uncertainty

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Introduction

Purpose

- Many situations require a decision where the values of outcomes are unknown
- However, proxies, or value signifiers, are often available to help evaluate the possible outcomes

Examples of Decision Making by Proxy

Car Resale Value

- Imagine you are buying one of two cars:
  - Mini Cooper
    - Dealer price: $16,500
  - Kia Spectra
    - Dealer price: $16,300
  - How much will they be worth in five years?
- Available proxies
  - Retail Price
    - Similar: Mini- $16.5K vs. Kia- $16.3K
  - Residual Value Estimates (ALG)
    - Very different: Mini- 44% vs. Kia- 15% (3:1!)
  - Will the Mini really be worth three times more than the Kia in 5 years? Which proxy is more reliable?

Antiquing

?

- Imagine buying one antique table versus another:
  - Sticker prices may or may not be indicative of the items' values
    - That is, the sticker price has an unknown reliability
  - So how is the actual monetary value of table derived from its proxy?

?

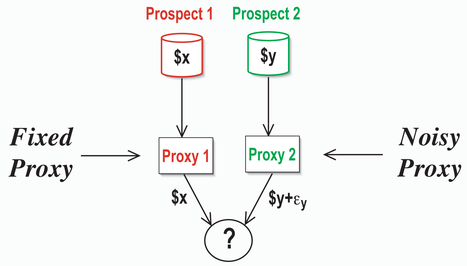
Research Questions

- When proxies act as decision intermediaries, how is the value of an item determined via a proxy?
  - *Face-value, Inference, Experience?*
- Does the reliability of the proxy affect the decision-making process?
  - *That is, does uncertainty about the relationship between proxy and the actual value matter?*
- How are decisions affected when proxy values are the only available indicator of variable monetary payoffs?
- In particular, does a prospect delivered through a noisy proxy change its decision value?

Methods

What if one proxy is less reliable than another?

The game can be manipulated to make one proxy noiser than the other



Details

Prospect Pairs

- Some specific prospect pairs were pre-selected to repeatedly occur
- For each of those prospect pairs, all values were held constant except x2
- x2 was raised/lowered after each trial depending on the subject's selection
- Eventually an x2 value was found which caused the subject to be indifferent between the two prospects

Selecting Prospect Pairs

- After each subject decision, the next prospect pair was randomly chosen from the set of pre-selected pairs and a random pair
  - Weighted so that random pairs occurred more frequently
  - Constrained so that random pairs must occur between pre-selected pairs

Selecting Prospect Pairs

- The pre-selected prospect pairs were mixed together with random prospect pairs
- Prospects in a random pair could have different bust probabilities
- Prospects in a random pair had roughly the same expected value

Finding the Indifference Point

- In these prospect pairs, the probability of a bust was the same for each prospect
- Indifference points (P=0.5, x2) were tracked using accelerated stochastic approximation (Kesten,1958)

$$Y_{n+1} = Y_n - \frac{c}{2 + m_{rev}}(R_n - 0.5),$$

Y_n is the n^{th} variable point value in $P2$
R_n is the n^{th} choice $P1$ vs. $P2$
m_{rev} is the number of reversals

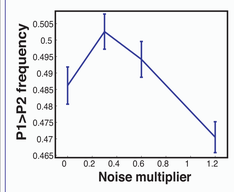
Selecting Exchange Rates

- After all indifference points had been found, the subject restarted the game in two new countries (with new exchange rates)
- The top country has an exchange rate with a noise level from the set {0.0, 0.3, 0.6, 1.2}
- The bottom country always has an exchange rate with zero noise
- Indifference points and other player data could then be compared among different noise levels

Results

Do overall *preferences* for proxies change?

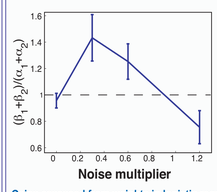
Frequency of selected P1 (noisy) > P2 vs. noise



Small effect, does not match predictions

Does overall *value* for proxies change?

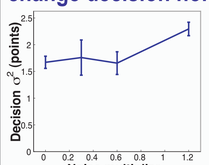
Average gain of prospect values P1 vs. P2



Gains assessed from weights in logistic regression analysis, and have the meaning of the slopes of the value function curves at the average prospect.

Suggests that Δpref due to change in value.

Does proxy noise change decision noise?



Slope param. (sensitivity) from Logistic reg. can be converted into an internal variance estimate

Changes in decision noise are not sig., until noise>1 (loss possible)

Analysis

Given prospects

$\theta_1 = (p_1, x_1; q_1, y_1)$ and $\theta_2 = (p_2, x_2; q_2, y_2)$

Assume decision are made by evaluating the sign of

$\Delta V_i = V_i(\theta_1) - V_i(\theta_2) + n_d$

Where

$V_i(\theta_1) = w(p_1^i)v(x_1^i) + w(q_1^i)v(y_1^i)$ *Prospect theory*

$V_i(\theta_1) = w(p_1^i + q_1^i)v(x_1^i) + w(q_1^i)(v(y_1^i) - v(x_1^i))$ *CPT*
$= v(x_1^i) + w(q_1^i)(v(y_1^i) - v(x_1^i))$ *in our study p+q=1*

$$v(x) = \begin{cases} x^{\alpha} & \text{Typical reported values} \\ -\lambda(-x)^{\alpha} & \alpha = 0.88,\ \lambda = 2.2 \end{cases}$$

$$w^+(p) = \frac{p^{\delta}}{\left(p^{\delta} + (1-p)^{\delta}\right)^{1/\delta}} \quad \text{Typical reported values} \quad \delta = 0.62 \text{ for gains}$$

Data Fitting

Assume

$$p(R=1|\Delta V) = \frac{1}{1+e^{-a\Delta V}}$$

$\Delta V_i = V_i(\theta_1) - V_i(\theta_2) + n_d \qquad E[n_d] = 0, E[n_d^2] = 1/a$

We fit a reduced model to this using Logistic regression

Results

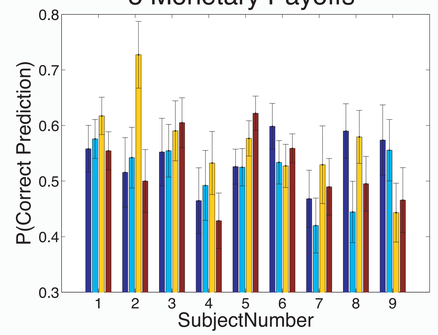
Do people predict money from proxy value based on recent trials?

Money values predications from recent history

Decisions predicted using Prospect theory on money

Trial by trial response prediction success measured

Choices Predicted from last 5 Monetary Payoffs



Not much evidence from methods used.

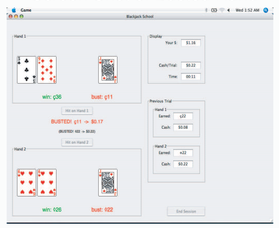
Methods

Blackjack School

We used a card game akin to blackjack to study decision-making by proxy

Screen Shot of Non-busted Hand

Screen Shot of Busted Hand



The Task

- Subjects were forced to hit on one of two hands
- Busted hands yielded x points while non-busted hands yielded y points

Proxy vs. Value

- Both hands were being played "online" in different countries
- Point values for each hand represented that country's foreign currency
- Foreign currency was translated into U.S. dollars after each trial/set of hands

The Manipulation

- Different currencies had different exchange rates and levels of stability
- $\$ = (m + \Delta m)/100$
  - $ - amount earned on a trial in U.S. dollars
  - m - amount earned in foreign currency
  - Δ is on interval [-noise, noise]
- Subjects earnings per condition:
  - *total = $/trial * minutesPlayed*

Hands = Prospects

- Each set of hands was a prospect pair: – (p_1, x1 ; q_1 , y_1) ; (p_2, x2 ; q_2, y_2)
- P and q were determined by the probability a hand would bust
- X and y were each hand's potential earnings in the foreign currency (points)

Predictions

Predictions

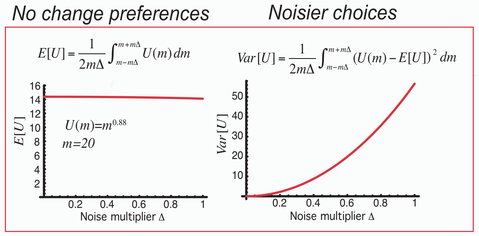
If payoff uncertainty via proxy = certainty effect
P1 ↓

If proxy uncertainty like ambiguity aversion
P1 ↓

If proxy to money conversion ignored
P1 unchanged

If proxy to money conversion is predicted based on recent experience (in the limit):

No change preferences *Noisier choices*



Results

Effects of noisy proxy on particular Prospect pairs

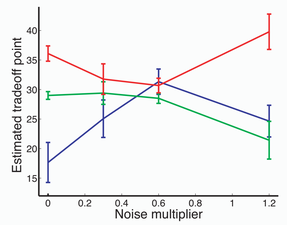
Δpref tracked by indiff. points using a trade-off method

If P1 (noisy) worth less indifference point ↓

Indifference Prospects:
(p, x ; q, y)

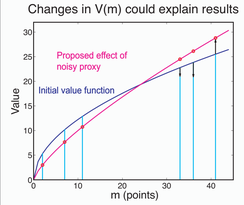
P1: Noisy Proxy
P2: Noiseless Match

	p	x	q	y
P1	0.46	2	0.54	41
P2	0.46	16	0.54	?
P1	0.38	7	0.62	33
P2	0.38	14	0.62	?
P1	0.31	11	0.69	36
P2	0.31	22	0.69	?



Clearly not true for all pairs what is going on?

Changes in V(m) could explain results



Change in V(x) shape can produce simultaneous increases and decreases

Indifference Point Analysis Details

Indifference prospects were fit with the following model: Assume Cumulative prospect theory,

$v(y_2) \approx v_2(y_1) + w(q)(v(x_1) - v_2(x_2)) + \varepsilon_{decision}$

Expanding this equation in a first order Taylor series around the expected midpoints

$r_1 = E[(x_1 - x_2)/2] \qquad r_2 = E[(y_2 - y_1)/2]$

We have

$y_2 - y_1 \approx w(q)\left(\frac{r_1}{r_2}\right)^{1-\alpha}(x_1 - x_2) + \kappa + \varepsilon_{decision}$

Which simplifies to

$dy \approx a\,dx + \kappa + \varepsilon_{decision}$

Motivating the decision model

$$P(R=1|dy, dx, a, s) = \frac{1}{1 + e^{-s(dy - a' dx)}}$$

Using Bayesian methods, we compute the marginal posterior distribution,

$P(a'|\{R_i, dy_i, dx_i\})$

Use to compute E[a'] and confidence intervals

Conclusions

Proxy uncertainty changes decision behavior: noise is not ignored for most subjects

Noisy proxies not worse than non-noisy: -simple predictions from previous effects seem to fail

Decision noise largely unaffected by proxy noise and local monetary tracking a poor predictor of choices

Detailed analysis supports a change in the shape of the value function for noisy proxy values

However, data are preliminary and more subjects are needed

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