A COMPARISON OF HUMAN AND IDEAL PERFORMANCE FOR THE DETECTION OF VISUAL PATTERN

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Chapter 4: Aspects of Phase in Visual Detection

Abstract. In three experiments, the role of phase information was investigated for the detection of gratings in the presence and absence of visual noise. There were four signal conditions: a .5 c/deg grating in noise, a 2 c/deg grating in noise, a .5 c/deg grating drifting at 6 Hz in noise and a .5 c/deg grating in the absence of noise. The gratings were Gaussian-enveloped in space and time. The extent of the gratings was one cycle between 1/e points of the spatial envelope.

For the "phase-uncertainty" experiment, thresholds were measured under two conditions. In the first condition, the observers were shown the phase of the grating prior to the trial. In the second, the phase was randomized from trial to trial so that the subject never knew what phase to expect. Prior knowledge of phase was shown to benefit the detection of the low spatial frequency (.5 c/deg) stationary grating in noise. If the spatial frequency was raised to 2 c/deg, prior knowledge of phase improved performance, but not as much for the .5 c/deg grating. If the .5 c/deg grating was drifted at 6 Hz, or if the noise was not present, prior knowledge of phase did not improve detectability.

In the second experiment, psychometric functions were measured for the detection of stimuli used in the first experiment. The psychometric functions for the .5 and 2 c/deg gratings in noise were shallower than would be expected from envelope, energy or channel uncertainty ($M>10$) models of visual detection. However, when the .5 c/deg grating was drifted in noise, or was detected in the absence of noise, the psychometric function slopes were significantly steeper than for either the stationary .5 or the 2 c/deg grating in noise.

In the third experiment, phase recognition (0 vs. 180° relative to a fixation mark) was measured with detection. Phase was more frequently recognized at threshold for the .5 and 2 c/deg gratings in noise than for the other two stimuli.

It is concluded that observers can have different detection strategies depending on signal conditions. These strategies resemble a single channel cross-correlator for the .5 and 2 c/deg grating in noise. For the .5 c/deg grating in the absence of noise and the .5 c/deg drifting grating in noise, the detection strategy resembles that of an energy detector or an ideal observer which is expecting more than 10 orthogonal signals.

INTRODUCTION

Phase information is clearly important in vision. For example, the amplitude spectrum carries no information about the distance between two identical, but separate, bright spots on a blank field. This information is carried by the phase spectrum. In the spatial frequency analysis of visual contrast detection, phase has received less attention than spatial frequency. This is perhaps due to the observation that variation of phase as a parameter does not usually affect results. Graham and Nachmias (1971) showed that relative phase has little effect on the detection threshold of the sum of two low contrast gratings with sufficiently different spatial frequencies. I have reported a similar result for gratings in the presence of noise (chapter 3). Threshold elevation following adaptation to a grating is independent of the phase between test and adapting grating, even when the retinal image is stabilized (Jones and Tulunay-Keesey, 1980). On the other hand, the relatively small adaptation to a spatial impulse suggests phase specificity (Legge, 1976), and recognition thresholds for judging the polarity of an edge or a thin bar are only slightly smaller (.05 log units) than detection threshold (Tolhurst and Dealy, 1975). Although it is undisputed that phase is an important parameter in suprathreshold vision, many aspects of the role of phase in high-contrast processing are yet unclear.

One way of exploring the limits to high-contrast perception is to superimpose the pattern of interest on "white" visual noise. Increasing noise levels increases the contrast thresholds for a given stimulus. Further, a specified noise presents computable limits to ideal performance. The ideal is determined by the detection task. Comparison of human performance with that of an ideal provides useful information about the mechanisms or strategies employed by the human observer on an absolute scale. Several ideals are described below.

Although the primary parameter of interest in this chapter is phase, it is important to place its role in detection in context. Peterson, Birdsall and Fox (1954) describe, among others, four classes of ideal detectors.

1) The cross-correlator is an implementation of the ideal when the signal is "known exactly" (Green and Swets, 1974). It is a coherent detector by which is meant that it makes use of phase.

2) The envelope detector is an implementation of the ideal for which all information, except the absolute phase (relative to an envelope), is available to the detector. It is also known as the "signal-known-exactly-except-for-phase" ideal. The envelope detector is sometimes referred to as an incoherent detector (see below).

3) The one-of-M-orthogonal-signals detector is the ideal for the case when any one of M orthogonal signals is expected, but it is not known which one. This ideal has been incorporated into a "channel-uncertainty" model of visual detection (Pelli, 1981). The

ideal can either be presumed to "know" the phase or not. Because for large M, there is very little difference in performance when phase is certain or uncertain, only the case for known phase will be considered.

4) The energy detector is an implementation of the ideal observer for noise signals, i.e. when only the spatial frequency spectrum and location of the signal are known. The energy detector is an incoherent detector in that it makes no use of phase information. All of these detection strategies will be considered in this chapter.

Phase Uncertainty Experiment (Exp. 1)

There are several ways to investigate which, if any of these strategies apply to human detection. A very direct way is to give the human observer the same visual task that the ideal of interest is designed to be best at. This has been done by Burgess, Wagner, Jennings and Barlow (1981) for the cross-correlator, and by Kersten (chapter 5) for the energy detector. Lasley and Cohn (1981) and Pelli (1981) have used this technique for the one-of-M-orthogonal signals ideal, but did not look at absolute performance. This method was used in the first experiment by pitting the human observers, in a detection task, against the cross-correlator and then against the envelope detector. Green and Swets (Fig. 7-6, p. 194) plot the percent correct in a two-alternative forced-choice task for these two detectors as a function of the signal-to-noise ratio. From their graph it can be seen that there is about a 6 db increase in signal-to-noise ratio (i.e. a factor 2 in contrast) required by the envelope detector to match 70% correct for the cross-correlator. Although this is very direct way of investigating the role of phase in detection, the predicted effect of phase uncertainty on performance is rather small. Thus two more experiments were done.

Psychometric Function Experiment (Exp. 2)

Another, but less direct way to study detection strategy, is to measure the psychometric function slope for a signal, under conditions which are not necessarily tailored to the ideal in question. For example, the slope of the psychometric function for the ideals mentioned above gets steeper as uncertainty increases (Green and Swets, 1974; Pelli, 1981). If steep psychometric functions are found for a specific signal, we might hypothesize that the observer was expecting any one of many signals. This is what Pelli (1981) has called "intrinsic uncertainty". The second experiment measured psychometric functions for the stimulus conditions described above.

When the psychometric functions in Fig. 7-6 of Green and Swets (p.174) are plotted in terms of $\sqrt{2}$ times the normal deviate of the proportion correct ($\sqrt{2}Z$) vs. contrast ($\underline{c}$) on double logarithmic coordinates, the cross-correlator has a slope (_k_) of 1 (dashed line in Fig. 4.1):

$$\log(Z) = \log(c) + \text{constant}$$

and the envelope detector a slope of 1.6 (dotted line in Fig.4.1):

$$\log(Z) = 1.6\log(c) + \text{constant}$$

Also shown in Fig. 4.1 is an upper bound on the performance for an energy detector, which has a slope of 2 in these coordinates (solid line) (see Appendix 4.1). Pelli (Fig. 5.3a, 1981) plots psychometric functions for the one-of-M-orthogonal-signals ideal for several M in the same coordinates. The psychometric function for M=10 differs from the envelope detector by at most 15% over most of the range shown in Fig. 4.1. Fig. 4.1 shows ideal performance for M=1000 from Pelli's graph (alternate dots and dashes). In this case _k_=3:

$$\log(Z) = 3\log(c) + \text{constant}$$

There are two trends to notice in Fig. 4.1. One is that as uncertainty (M) increases the slope _k_ increases.

Secondly, relative to the cross-correlator, performance at a given signal-to-noise ratio (or contrast, given a fixed noise level) declines as M increases. The psychometric functions shown in Fig. 4.1 not only show the relative slopes but show absolute performance for a specific stimulus condition used in this study (see Appendix 4.1). For example, for a given signal-to-noise ratio, a one-of-10-orthogonal signals detector can not do better than the

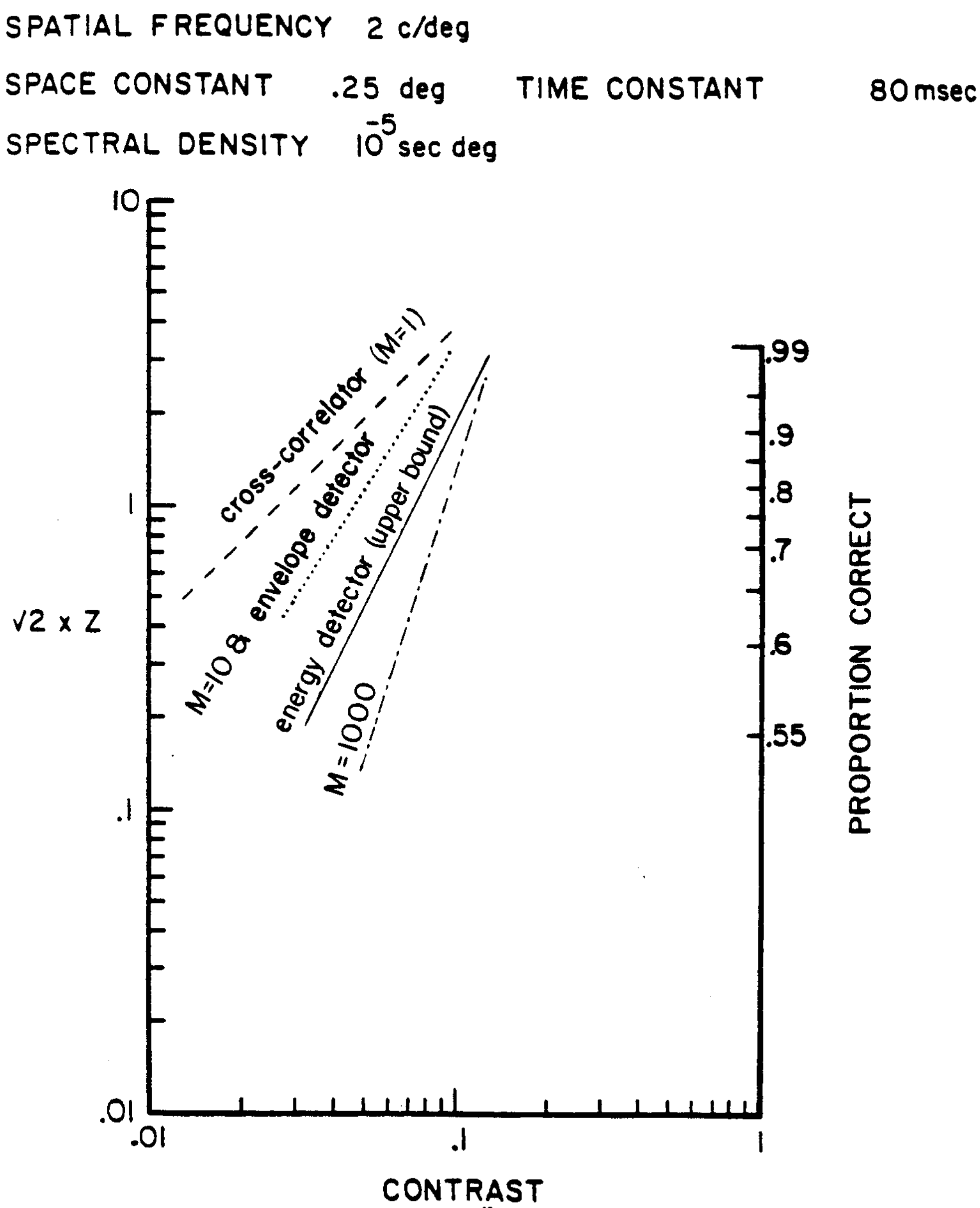

Fig. 4.1. "Psychometric functions" plotted as log √2Z vs. log contrast for the cross-correlator, envelope, energy and one-of-1000-orthogonal signals detectors for a Gaussian enveloped sinusoid in noise (stimulus conditions specified at the top of figure). The psychometric function, shown for the energy detector, is an approximation which represents an upper bound on performance. The one-of-M-orthogonal signals detector is ideal when the M signals are "known". The cross-correlator is equivalent to the M=1 detector. Performance of the envelope detector is approximately equal to the detector with M=10 (to within 15%), when the 10 signals are in fact "known".

appropriate one-of-1-orthogonal signals detector (i.e. labelled cross-correlator in Fig. 4.1) when there is only one signal to be detected.

Phase Recognition Experiment (Exp. 3)

A third way to investigate possible detection strategies, is to measure performance for some other task which nevertheless has some bearing on the model being considered. For example, one might argue that if the observer is making use of phase information to decide whether a signal is present or not, the knowledge of what the phase is, might be available too. Thus, in the third experiment, the observer was required to detect the signal as well as recognize its phase relative to the fixation mark.

Stimulus Conditions

In this study, four stimulus conditions were chosen which suggest transitions between an ability and inability to use phase information in detection. The stimulus conditions were:

1) A spatially narrowly windowed 2 c/deg grating (about one cycle wide) in dynamic visual noise. It was hypothesized in chapter

2 that detection of gratings in noise showed properties of cross-correlation. Utilization of phase information is inherent in cross-correlation (see below). A spatially narrow window was chosen because previous measurements indicated that efficiencies are highest for gratings in noise of one cycle's width (chapter 2). Because of the high efficiency with which this pattern is detected (10-30%), particular attention will be paid to absolute performance with respect to two ideal observers, the cross-correlator and envelope detectors.

2) A .5 c/deg grating (about one cycle wide) in the absence of noise. Several studies have implicated phase in detection of patterns against a blank screen. In particular, Campbell, Johnstone and Ross (1981), following some earlier evidence of McCann, Savoy, Hall and Scarpetti (1974), have suggested that luminance gradient rather than spatial frequency is an important determinant of threshold for gratings of spatial frequency 1 c/deg or less. They point out that gradient sensitivity implies phase sensitivity. Also Stromeyer, Klein, Dawson and Spillman (1982) have shown phase-selective adaptation to .5 c/deg gratings. Thus we might expect phase to play a relevant role in the detection of a .5 c/deg grating in the absence of noise.

3) A .5 c/deg grating drifting (at 6 Hz) through one cycle in the presentation time. Preliminary measurements showed that efficiency dropped when gratings were drifted sufficiently fast in

noise (e.g. one period in one presentation time). The loss in efficiency might be due to an inability to use phase information for rapidly drifting gratings in noise.

4) A stationary .5 c/deg grating in noise. Measurements were made for comparison with the previous conditions with respect to parameters of spatial frequency, noise level and drift rate. Further details concerning the stimuli are in METHODS.

METHOD

Apparatus

The signal patterns were sine-wave gratings with Gaussian spatial and temporal windows. The patterns were produced on the face of a Joyce Electronics CRT display by Z-axis modulation (Campbell and Green, 1965). The display was of the electromagnetic deflection type, with a raster frequency of 100 kHz, and a non-interlaced frame rate of 100 Hz. The display had a P31 phosphor, an unmodulated luminance of 340 cd/m^2, and a dark surround. The viewing distance was 228 cm. The display was 30 cm wide and 16 cm high. Thus, at the viewing distance of 228 cm, the screen subtended 7.5^o horizontally by 4^o vertically. Photometric calibrations were conducted with an UDT 80X Opto-meter.

The signal luminance waveforms were synthesized digitally by an LSI-11/23 computer. In each 10 msec frame, the computer generated 300 pairs of voltage samples which were routed through two 12-bit digital-to-analog converters (DAC). One DAC generated the spatial envelope, and the other the sinusoidal grating. The outputs of the two DACs were then multiplied and routed through a programmable dB attenuator which temporally modulated the signal voltage which was subsequently added to the noise.

The luminance noise varied in time and one spatial dimension. Pseudorandom noise was digitally synthesized by a 31-bit shift register with exclusive-or feedback (Roberts, 1963; Horowitz and Hill, 1980; Pelli, 1981). To produce Gaussian one-dimensional noise, the shift register was clocked at f_c and low-pass filtered at the .1 dB downpoint, .1 x f_c. The bandwidth of the noise was always kept flat to 22 c/deg and 50 Hz. At the maximum rate used (f_c=200 kHz), the noise generator would cycle once ($2^{31}-1$ shifts) in 6 hours.

Stimulus Conditions

The luminance of the signal at a point $\underline{x}$, $\underline{y}$ (in degrees) at time $\underline{t}$ (sec) is given by:

$$L(x,y,t) = L_o \times (1 + m(x,t)s(x,t)) \times \mathrm{rect}(y/b),$$

where $\underline{L_o}$ is the mean luminance. The term $\underline{s}(\underline{x},\underline{t})$ is given by:

$$s(x,t) = c_m \cos(2\pi(fx-ht) + \theta),$$

where $\underline{f}$ and $\underline{h}$ are the spatial and temporal frequencies respectively. θ is the phase relative to the fixation mark. $\underline{c}_m$ is the Michelson contrast prior to windowing. rect(y/b) is a rectangle function where $\underline{b}$ is the vertical height in degrees and equaled the vertical height of the screen. The modulating function $\underline{m}(\underline{x},\underline{t})$ is given by the product of Gaussian functions horizontally and temporally,

$$m(x,t)=\exp(-(x/s_x)^2-(t/s_t)^2)$$

where $\underline{s}_x$ and $\underline{s}_t$ are the space and time constants.

There were four signal conditions used in these experiments: a .5 c/deg grating in noise, a 2 c/deg grating in noise, a .5 c/deg grating drifting at 6 Hz in noise and a .5 c/deg grating in the absence of noise. The temporal envelope was Gaussian with a time constant of 80 msec. The spatial windows were Gaussian, one cycle wide between 1/e points. This corresponds to 1.7 octaves width at half height. The contrasts reported are the Michelson contrasts prior to applying the spatial or temporal windows.

Percent correct was measured in a two-alternative forced-choice procedure. The intervals were separated by 600 msec. Each interval was marked by an auditory tone. A feedback tone indicated to the observer whether he was right.

The noise spectral density was either 0 (no noise) or 10^{-5} sec-deg. The noise was turned abruptly on 80 msec before the first temporal 1/e point of the signal was reached, and turned abruptly off

80 msec after the second 1/e point. The noise extended uniformly across the screen.

Procedure: Phase-Uncertainty Experiment

For the phase-uncertainty experiment, the proportion correct for detection of a signal of fixed contrast energy was measured as a function of phase angle under two conditions. Phase was defined relative to a small (1' in diameter) black fixation mark.

In one condition, the observer was given a block of two-interval forced-choice trials in which the signal always had the same phase. This will be referred to as the phase certain case. To minimize uncertainty, a sample of the signal was shown prior to each trial and the same signal was used in all trials in a given block. When detecting in noise, the sample signal was shown at the same contrast as the trial signal in the absence of noise. For the no noise conditions, the signal was shown at a 40% greater dB level than the trial signal contrast.

In the second condition, the phase was randomized from trial to trial, eliminating any opportunity to benefit from knowledge of phase in the detection task. This condition will be referred to as the phase uncertain case. For the 2 c/deg grating, there were 8 phases spaced 45° apart. For the .5 c/deg gratings, proportion correct was measured at each of 20 phases equally spaced between -180 and 180°.

Using 8 and 20 phases for the 2 and .5 c/deg conditions permitted the minimum phase differences, in terms of visual angle, to be close: 3.8' and 6' for the 2 and .5 c/deg gratings respectively. For the widths used, a constant contrast (before windowing) does not change signal energy by more than 1.5% across phase.

For some signal conditions, performance was measured at two contrasts. For these cases, psychometric function slopes were computed for both the phase-certain and -uncertain data, in order to find out whether phase-uncertainty affected the slopes. There were at least two sessions of 600 trials each for both conditions. Two sessions were run in a day. On the second day, the order of the conditions was reversed.

The following procedure was used to derive a measure of the effect of uncertainty and to assess the reliability of the difference between the proportions correct, p_c and p_u, in the phase-certain and -uncertain conditions. The binomial distribution of the proportion correct is well approximated by a normal distribution for many samples (Lindgren, 1976, p. 173). The estimate of the variance is $p_i(1-p_i)/n$, where p_i is the proportion correct for the i^{th} condition and n the number of trials. The difference between the means divided by the square root of the sum of the variance estimates (x) was used as the test statistic to compare two proportions correct:

$$x = \frac{(p_c - p_u)\sqrt{n}}{\sqrt{\{p_c + p_u - p_c^2 - p_u^2\}}}$$

Let $\underline{C}$ be the normal deviate of $\underline{x}$:

$$C = Z(x).$$

If $\underline{p}_c$ and $\underline{p}_u$ are the estimates of population means $\underline{\rho}_c$ and $\underline{\rho}_u$, then $\underline{C}$ can be interpreted as the estimated probability that $\underline{\rho}_c$ is less than $\underline{\rho}_u$. A summary parameter $\underline{C}'$, was defined as the average of the C's taken across psychometric functions and observers for a given stimulus condition. $\underline{C}'$ can be roughly interpreted as an estimate of the probability that the proportion correct is greater under the phase-certain than -uncertain condition. However, rather than trying to use C' as in a test of "significance", it serves the purpose of placing the effect of uncertainty on a continuum in which values of C' close to 0 indicate that prior knowledge of phase benefits performance, and values close to 1 which indicate that prior knowledge of phase hinders performance. No effect of prior knowledge would have values of C' close to .5.

Procedure: Psychometric Function Experiment

For the psychometric functions, a two-alternative forced-choice detection procedure was also used. The percent correct was measured in blocks of 100 to 130 trials for each contrast. There were five blocks at five contrasts per session. The signals were in cosine-phase with respect to the fixation mark. As above, to minimize uncertainty, a sample of the signal was shown prior to each trial. All psychometric function slopes are based on results combined across two or more sessions and a minimum of 1000 trials.

The dependent variable of interest is the steepness, $\underline{k}$ of the psychometric function. When $\sqrt{2}$ times the Z-score ($\underline{Z}$) of the proportion correct is plotted against contrast $\underline{c}$ on log-log coordinates, $\underline{k}$ represents the slope:

$$\log(Z) = k\log(c) + \text{constant}$$

The slopes were computed using a maximum likelihood procedure originally developed by Watson (1979) and modified by Rubin (1982) to fit a Gaussian rather than Weibull distribution. The 95% confidence intervals for the slopes (indicated in parentheses in Table 4.3) were based on Monte Carlo simulations assuming binomial statistics. 500 simulations of each psychometric function were used to estimate the standard deviation of the slope. In most cases a log-normal density for the slopes produced a better chi-square fit than did a normal density. The 95% confidence interval was calculated by multiplying the estimated slope by $10^{\pm 1.96\gamma}$, where $\underline{\gamma}$ is the standard deviation estimated from the Monte Carlo runs. On average, the estimated 95% confidence intervals for a session of 1000 trials were about ±37%.

Procedure: Phase-Recognition Experiment

For the phase-recognition experiment, in a given trial the signal was either at a phase angle of 0° or 180°. That is, there was either a bright or a dark bar behind the fixation mark. For the drifting grating, the bright and dark bars appeared behind the fixation mark in the middle of the temporal interval for the 0° and 180° phases. The observer was asked to choose which of the two intervals the signal occured in and whether it was 0° or 180° phase. 200 trials were presented for each of two contrasts for each signal condition.

Tanner (1956) defined what might be thought of as a measure of the "internal correlation" of two signals based on the proportions correctly detected and recognized. The following formula was adapted from Tanner's. The correlation angle θ was defined as:

$$\cos(\theta) = 1 - (Z_r/Z_d)^2$$

where Z_r and Z_d are the Z-scores of the proportions correctly detected and recognized. This formula is equivalent to Tanner's when the detectabilities (d') of the two signals are equal. (For the stimuli used here, the detectabilities were not reliably different.) For an ideal observer (signal-known-exactly), two orthogonal signals have a correlation angle of 90°. If the two signals are identical, then θ is 0. If two signals are anti-correlated (as is the case in this experiment), θ is 180^{0}.

Observers

There were three observers. DK and DR are emmetropes. KJ has one emmetropic eye, the other was 1.25 D myopic and was left uncorrected. DK is the author and took part in all the conditions. All observers, except for DK were naive to the details of the experiment and its motivations. Viewing was binocular with natural pupils.

RESULTS and DISCUSSION

A few comments concerning the presentation of the results may be useful. The results of the three experiments are summarized primarily in four tables: Tables 4.1 and 4.2 for the phase-uncertainty experiment, Table 4.3 for the psychometric function experiment and Table 4.4 for the phase-recognition experiment. Several graphs for each experiment show examples of some results to aid in understanding what was tabulated.

Phase Uncertainty Experiment

We will examine several aspects of the phase-uncertainty results. The principal question is whether randomization of phase leads to a decrease in performance. An additional point is whether an increase in psychometric function slope might accompany phase uncertainty. This would be predicted if the observer changes detection strategy from that of a cross-correlator to that of an envelope detector. However, before addressing either of these two issues, the dependence of performance on the phase angle is examined.

Performance as a function of phase. Fig. 4.2 shows the proportion correct for detection of a .5 c/deg grating in noise with space constant of 1° for DK. The contrast was 8%. The filled symbols represent data for the phase-certain case. The open symbols represent data for the phase-uncertain case. The error bars indicate 95% confidence intervals based on binomial statistics. The upper and lower portions of the 95% confidence interval were given by $\pm 1.96 \times \sqrt{p(1-p)/n}$, where p is the proportion correct and n the number of trials. Each point on the graph represents the proportion correct for data pooled into bins centered at phases of 0, 45, 90, 135 and 180 degrees for positive and negative phase angles. Each point is based on about 240 trials.

Because the signal energy was constant, efficiency (defined below) is monotonically related to the proportion correct in this figure. Both KJ and DK showed greater efficiency for the 0 and 180° phases over the 90° phase for .5 c/deg for certain and uncertain cases in the presence and absence of noise. For DK, the efficiency (relative to the signal-known-exactly ideal) was 9.8% for 0° phase and dropped to 3.0% for the 90° phase. Although a significant

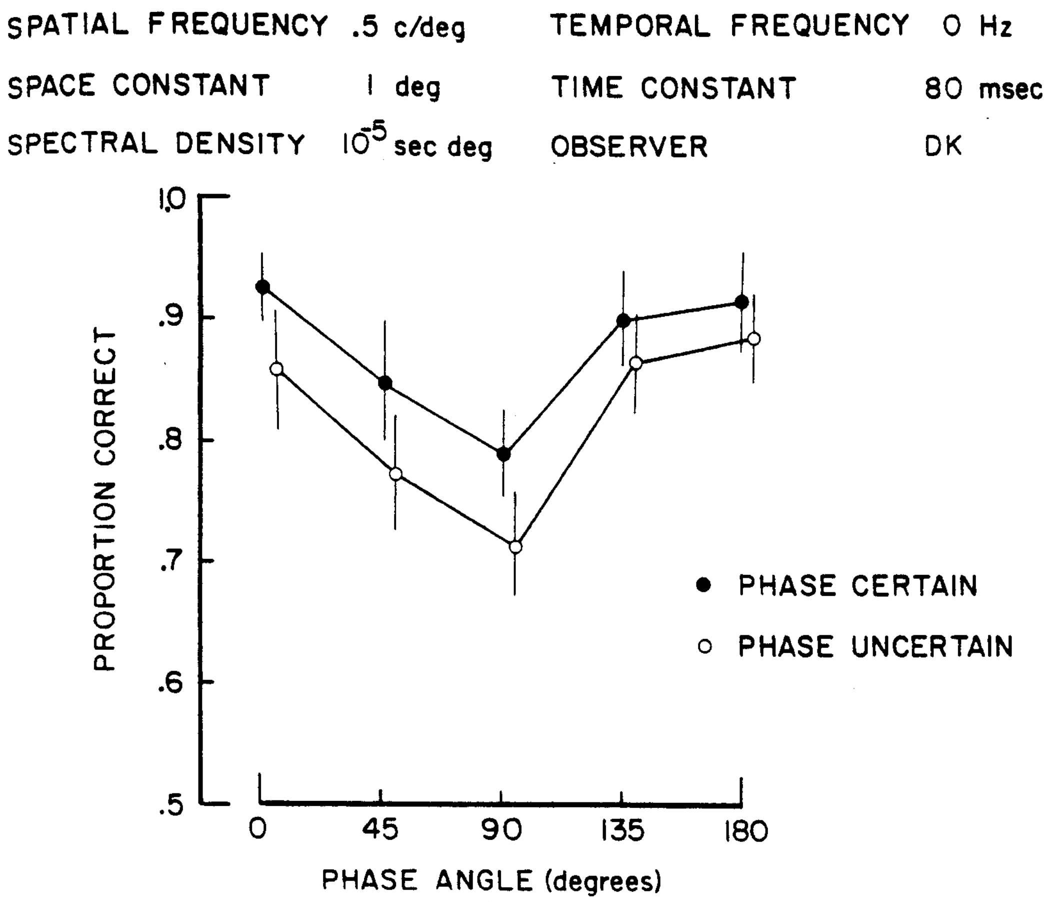

Fig. 4.2. Proportion correct as a function of phase angle relative to the Gaussian envelope for a .5 c/deg grating in the presence of noise (Exp. 1). The observer was either shown the phase to expect prior to a trial (filled circle) or not, i.e., the phase was randomized (open circles). The error bars represent 95% confidence intervals based on binomial statistics. Observer DK.

difference was found for some pairs of 0 vs. 90° or 90 vs. 180° for the 2 c/deg grating in noise and the .5 c/deg grating in the absence of noise ($p < .05$), it was not consistently found for all pairs. There was no reliable difference in sensitivity between phases for the drifting grating. Perhaps related to this finding, is the monocular rivalry phenomenon reported by Atkinson and Campbell (1974). They interpreted their findings by suggesting phase selective mechanisms responsive to only 0 and 180°.

Effect of uncertainty on performance. Table 4.1 summarizes the principal results of the phase-uncertainty experiment. The proportions correct averaged across phase are shown for the phase-certain (p_c) and -uncertain (p_u) cases. The right-most column gives the statistic C described above. The largest effect of phase-uncertainty is seen for the .5 c/deg stationary grating in noise. It will be shown below that this is a smaller effect than would be predicted if the observer changed from completely coherent detector, i.e. a cross-correlator, to an incoherent detector, e.g. an envelope detector. There is a smaller effect of phase randomization for the 2 c/deg stationary grating in noise. In contrast to these two conditions, there is no benefit of prior knowledge of phase for the .5 c/deg grating in the absence of noise or when it is drifting. If anything, the observers show a bias to do poorly when trying to use phase information.

The results can be summarized as follows. Using the parameter C', it is possible to order the stimulus conditions in order of decreasing benefit of prior phase information: the stationary .5 c/deg in noise (C' = .002 ±.002), the stationary 2 c/deg grating in noise (C' = .09 ±.05), the .5 c/deg grating drifting in noise (C' = .69 ±.17) and the .5 c/deg grating in the absence of noise (C' = .96 ±.02). These results suggest that the first two stimulus conditions involve coherent detectors, whereas the second two do not.

There is a direct way to compare the relevance of prior knowledge of phase for the stimulus conditions involving noise. The efficiency for the phase-certain and -uncertain cases can be calculated relative to the cross-correlator and envelope detectors respectively. If efficiencies are higher when phase is randomized than when phase is not, then the observer is "phase uncertain". The way to calculate efficiency can be easily understood in terms of Fig.4.1. For a given $\underline{Z}$, corresponding to a proportion correct, we can graphically find the contrast threshold which the ideal of interest would have ($\underline{c}_i$), and the contrast threshold which the observer had under the same condition ($\underline{c}_o$). The efficiency ($\underline{E}$) is the squared ratio of the ideal's contrast threshold to the observer's:

$$E = (c_i/c_o)^2$$

For the 2 c/deg grating in noise, the psychometric function for the cross-correlator is given by:

TABLE 4.1

Phase Uncertainty Data

Observer : DK

Spatial Frequency (c/deg)	Temporal Frequency (Hz)	Noise Spectral Density (sec deg)	Contrast (%)	Per Cent Correct Phase Certain $(100p_c)$	Per Cent Correct Phase Uncertain $(100p_u)$	Probability (C) $(\rho_c < \rho_u)$
.5	0	10^{-5}	8	87±2	78±2	2.6×10^{-9}
2	0	10^{-5}	10	86±2	85±2	.243
			8	79±3	74±4	.02
.5	6	10^{-5}	12	85±3	88±3	.94
			8	66±4	68±4	.77
.5	0	0	1.75	90±2	93±2	.996

Observer : KJ

.5	0	10^{-5}	8	88±2	83±3	.007
			4	73±4	63±4	9.6×10^{-5}
2	0	10^{-5}	12	89±2	87±2	.089
			5.1	69±4	63±4	.003
.5	0	0	2	80±3	84±3	.965
			1.35	59±4	63±4	.922

Observer : DR

.5	6	10^{-5}	11	76	75.3	.37

$$\sqrt{2Z} = 40c$$

and for the envelope detector by:

$$\sqrt{2Z} = 144c^{1.6}$$

These lines are shown in Fig. 4.1 by the dashed and dotted lines respectively.

The average efficiency (across both observers, phases and contrasts) for the 2 c/deg grating in noise for certain phase was 12.6% ±.53. For the phase-uncertain case, the average efficiency was 28.3% ±1.7. This approximate factor of two increase in efficiency can be interpreted as evidence for some intrinsic phase uncertainty. This is only half the increase in efficiency expected had there been no effect of phase uncertainty. (Recall that about a 6 dB decline in contrast threshold accompanies optimal phase utilization at the 70% correct point.)

One possible interpretation, for the lack of a large effect of phase-uncertainty for the 2 c/deg grating, would be that phase is coarsely sampled by the observer. Burr (1980) reports results that support such a conclusion. Burr had observers discriminate a change of phase of a 3f c/deg component relative to a fundamental _f_ c/deg. The contrast of the fundamental was three times that of the harmonic. For harmonic contrasts of 3% and 10%, Burr found a threshold of about 30° phase angle of a third harmonic relative to its fundamental over a range of harmonic frequencies from 1 to about 15 c/deg. It was impossible to discriminate phase for harmonics higher than 30 c/deg. This relates well to the observation that coarse quantization of phase is not very detrimental to image quality (Piotrowski, 1981) and suggests an explanation for the small effect of phase uncertainty. Burr (1980) concludes that phase discrimination is made by phase selective mechanisms, rather than by mechanisms sensitive to absolute change in retinal image position.[1]

There was no decline in performance for phase-certain vs. phase-uncertain cases when the .5 c/deg grating was drifted. In this case DK's efficiency rises by the predicted factor of about 4, from 1% to 4.6% for the 8% contrast for the phase-certain and phase-uncertain case.

DK's efficiencies for the stationary .5 c/deg grating at 8% contrast were 6.2% and 8.7% for the phase-certain and -uncertain cases respectively. The fact that the efficiencies are almost equal suggests a flexible detection strategy. It as if the observer is as "good" at cross-correlation as at envelope detection, (although the actual algorithms used by the observer may be unlike either

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[1] Although Burr's results are more applicable to the results found here, they can be contrasted with Westheimer's finding that thresholds for discriminating a sudden change in phase for high contrast gratings was about 1° phase angle per c/deg (Westheimer, 1978). This corresponds to a rather fine resolution of about 10'' of visual angle over a range of 3 to 25 c/deg.

cross-correlation or envelope detection). Note too that the efficiency, for the phase-uncertain case, drops by a factor of 2 or so when the grating is drifted. However, for the phase-certain case, the efficiency drops by a factor of 6. This observation is consistent with the hypothesis proposed earlier, that some of the efficiency lost when drifting a grating in noise can be accounted for in terms of phase uncertainty.

Phase uncertainty and the psychometric function slope. For each signal condition in the first experiment, data were collected at two points for at least one observer permitting the calculation of a psychometic function slope. Fig. 4.3 shows data taken from Table 4.1 for DK for the 2 c/deg grating in the presence of noise. Error bars indicate 95% confidence intervals. It can be seen that slope for the phase-certain psychometric function (filled symbols) is shallower than for the phase-uncertain one (open symbols). Table 4.2 summarizes the slopes (k) of the psychometric functions under the phase-certain and -uncertain conditions. For the .5 and 2 c/deg gratings in noise the phase-certain slope is shallower than the phase-uncertain slope (means for DK and KJ of 1.12 vs. 1.89). For the drifting grating, the slopes are equal for the two cases, and for the .5 c/deg grating in the absence of noise, the phase-certain slope is greater. Although it is difficult to attach significance to these findings by themselves, they are consistent with the other findings of this chapter. Psychometric function slopes of the stationary .5 and 2 c/deg gratings in noise resemble those of a cross-correlator when the phase is known, but become more like an envelope detector when the phase is randomized. However, detection strategy for the .5 drifting grating in noise, and the .5 stationary grating in the absence of noise is relatively inflexible with respect to prior knowledge of phase. It is as if observers are "intrinsically uncertain" regarding phase at detection threshold for these stimulus conditions. A related result has been reported by Lasley and Cohn (1981) who measured the steepening of the psychometric function when temporal uncertainty was introduced in the discrimination of a small (6.9') spot of light presented for 32 msec on a pedestal. Pelli (1981) has found a steepening of the psychometric function for detection vs. contrast discrimination of a narrow (.25°), brief (20 msec) bar at one of 10,000 places and times.

Psychometric Function Experiment

In the previous experiment, the psychometric function slopes changed for some stimulus conditions, depending on prior knowledge of phase. In the second experiment, the emphasis was on obtaining reliable estimates of the psychometric function slopes without the complications of additional tasks, conditions or phases. The phase information was always given prior to the trials. Table 4.3 summarizes the psychometric function data. It can be seen that, based on the 95% confidence intervals, the stimulus conditions can be

SPATIAL FREQUENCY 2 c/deg

SPACE CONSTANT .25 deg

SPECTRAL DENSITY 10^{-5} sec deg

TEMPORAL FREQUENCY 0 Hz

TIME CONSTANT 80 msec

OBSERVER KJ

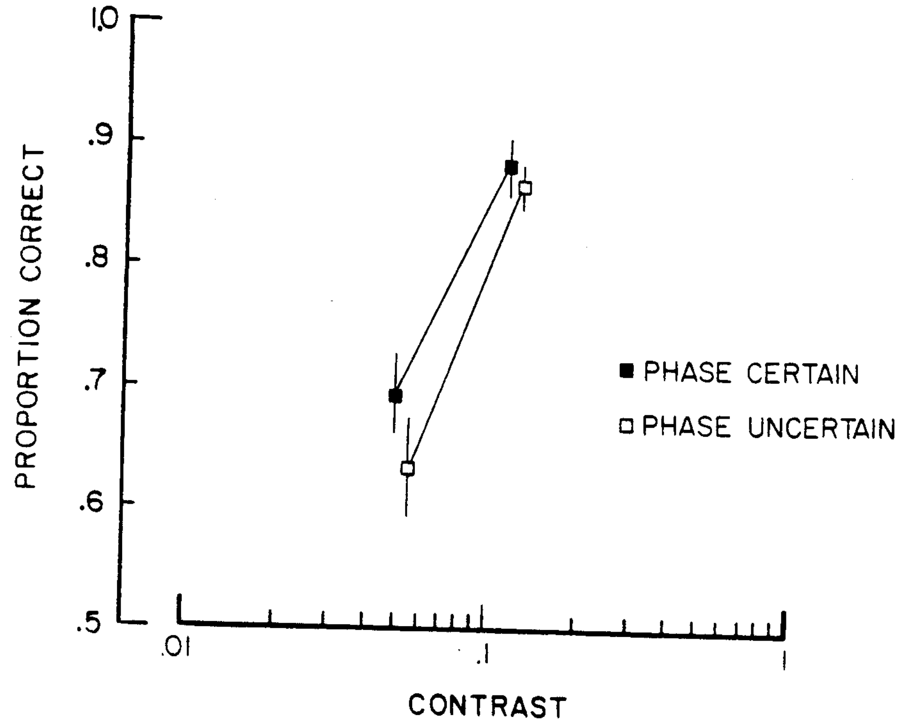

Fig. 4.3. Proportion correct as a function of contrast for a 2 c/deg grating in the presence of noise (Exp. 1). As for Fig. 4.2, the observer was either shown the phase to expect (filled squares) or the phases were randomized from trial to trial (open squares). The error bars represent 95% confidence intervals. Observer KJ. Data for the other conditions are shown in Table 4.1.

TABLE 4.2

Psychometric Functions from Phase-Uncertainty Experiment (Exp. 1)

Subject	Spatial Frequency (c/deg)	Temporal Frequency (Drift) (Hz)	Noise Spectral Density (sec deg)	Number of Trials	Slope (k)	Phase Case
KJ	.5	0	10^{-5}	1200	.95	certain
				1200	1.54	uncertain
DK	2	0	10^{-5}	1200	1.29	certain
				1200	2.23	uncertain
KJ	2	0	10^{-5}	1920	1.08	certain
				1920	1.43	uncertain
KJ	.5	0	0	1900	3.36	certain
				1200	2.75	uncertain
DK	.5	6	10^{-5}	1200	2.3	certain
				1200	2.3	uncertain

TABLE 4.3

Psychometric Function Data (Exp. 2)

Subject	Spatial Frequency (c/deg)	Temporal Frequency (Hz)	Width (deg)	Noise Spectral Density (sec deg)	Number of Trials	Slope (k)	95% Confidence Interval
DK	.5	0	1.0	10^{-5}	1200	1.15	(.83-1.59)
DK	2	0	.25	10^{-5}	1000	0.85	(.61-1.18)
DK	.5	0	1.0	0	1300	*2.34	(1.86-2.95)
DK	.5	6	1.0	10^{-5}	1000	*2.89	(2.17-3.84)
KJ	.5	0	1.0	10^{-5}	1817	1.12	(.87-1.45)
KJ	2	0	.25	10^{-5}	1000	1.10	(.84-1.44)
KJ	.5	0	1.0	0	1000	*1.67	(1.33-2.10)
DR	.5	6	1.0	10^{-5}	1200	*2.08	(1.68-2.58)

*These slopes are significantly different from one by the test described in the text.

partitioned into two categories: the conditions for which the slopes are not significantly different from 1, and those for which they are. The .5 and 2 c/deg stationary gratings in noise fall into the first category, and the .5 c/deg drifting grating in noise, and the .5 c/deg in the absence of noise fall into the second category. Detection for signals in the first category resembles cross-correlation. In the second category, the slopes are higher and resemble the incoherent detectors (energy or envelope) or the one-of-M-orthogonal signals (M>10) detectors. This categorization is consistent with the previous experiment.

Shallow psychometric functions have been measured for discrimination of gratings differing only in contrast (Nachmias and Sansbury, 1974). At least two explanations have been advanced to account for this finding. Nachmias and Sansbury (1974) originally suggested an accelerating non-linearity in the visual response. This idea has been incorporated into a successful model of spatial frequency masking by Legge and Foley (1980). However, as noted earlier in this chapter, it has also been suggested that channel uncertainty could be used to account for the steepening of the psychometric function when the masker is not present (Tanner, 1961; Nachmias, 1972; Cohn, Thibos and Kleinstein, 1974). (Foley and Legge (1981) discuss non-linear transducer vs. channel uncertainty model explanations of psychometric function slopes.) Uncertainty could involve many parameters other than phase. Either increased uncertainty or the introduction of a non-linearity could account for the steeper slopes found for the .5 c/deg grating in the absence of noise and the .5 c/deg grating drifting in noise.

If detection of stationary gratings is mediated by a single channel, then Birdsall's theorem could be used to argue that phase uncertainty accounts for the steeper psychometric function in the absence of noise (Lasley and Cohn, 1981). In brief, this theorem says that if a signal is to be detected in the presence of noise, then the introduction of a deterministic monotone non-linearity just prior to making the decision has no effect on performance. However, it could be argued that there is multiple pooling across many detectors in space and spatial frequency in the absence of noise (e.g. Graham, Robson and Nachmias, 1978). If this is the case, Birdsall's theorem is not applicable because it applies to a single channel. Further, as argued in chapter 2, detection of stationary gratings in noise may be mediated by a single channel. Thus, very different detection strategies may be employed for detection in noise vs. no noise. However, this very objection to the use of Birdsall's theorem suggests potentially greater intrinsic uncertainty for detection in the absence of noise. Perhaps the best support for an uncertainty interpretation of the slope of the psychometric function is that slopes were near one for exactly the two stimuli (the stationary .5 and 2 c/deg gratings in noise) for which observers benefited from prior knowledge of phase. The slopes for the other two signals (the stationary .5 c/deg grating in the absence of noise and the .5 c/deg grating drifting in noise) were significantly different from one, and observers did not benefit from prior knowledge of phase for these signals.

In addition to comparing the slopes of the psychometric functions, it is possible to compare human and ideal performance directly. We can exclude models of detection which perform at an inferior level compared with human observers. Because the efficiency of the 2 c/deg grating in noise is one of the highest found (see chapter 2), let us examine the absolute level of performance in detail. To help put the results in context, Fig. 4.4 shows psychometric functions (from Fig. 4.1) for the cross-correlator (*k*=1), the envelope detector (*k*=1.6) and the one-of-1000-orthogonal signals ideal (*k*=3). An approximate upper bound on an energy detector (*k*=2) is also shown. The signal is a 2 c/deg grating, with space and time constants equal to .25^o and 80 msec and is detected in white Gaussian noise of spectral density 10^{-5} sec-deg. Appendix 4.1 describes how these psychometric functions were obtained. The filled and open symbols show DK's and KJ's performances respectively. Because the lower set of data points for each observer fall above those of the energy detector, and the M=1000 ideal, these models can be ruled out on the grounds that human observers do better. This is independent of criticisms that the psychometric functions might be spuriously shallow due to non-stationarity of the mean contrast threshold (Hallett, 1969). If the psychometric function slopes are spuriously shallow for this reason, then a channel-uncertainty model with about 20 orthogonal channels might account for performance with the assumption of little or no additive internal noise.

Phase Recognition

Fig. 4.5 shows an example of a psychometric function obtained in the third experiment for DK. Both the .5 c/deg drifting grating in noise (shown) and the .5 c/deg grating in the absence of noise were detected more frequently than recognized at both contrasts for both observers. Table 4.4 summarizes the results for all the conditions. The rightmost column shows the correlation angle $\underline{\theta}$. In order of increasing correlation (decreasing $\underline{\theta}$), the stimulus conditions can be ordered as follows: the stationary .5 c/deg grating in noise ($\underline{\theta}$ = 100° ±13), the stationary 2 c/deg grating in noise ($\underline{\theta}$ = 91 ±7), the .5 c/deg grating in the absence of noise ($\underline{\theta}$ = 44±9) and the .5 c/deg grating drifting in noise ($\underline{\theta}$ = 21±6). The small correlation (larger angle $\underline{\theta}$) found for the first two stimulus conditions suggests that phase is more important for the visual processing (and perhaps detection) of these two signals than for the second two. This categorization is consistent with results from the previous two experiments.

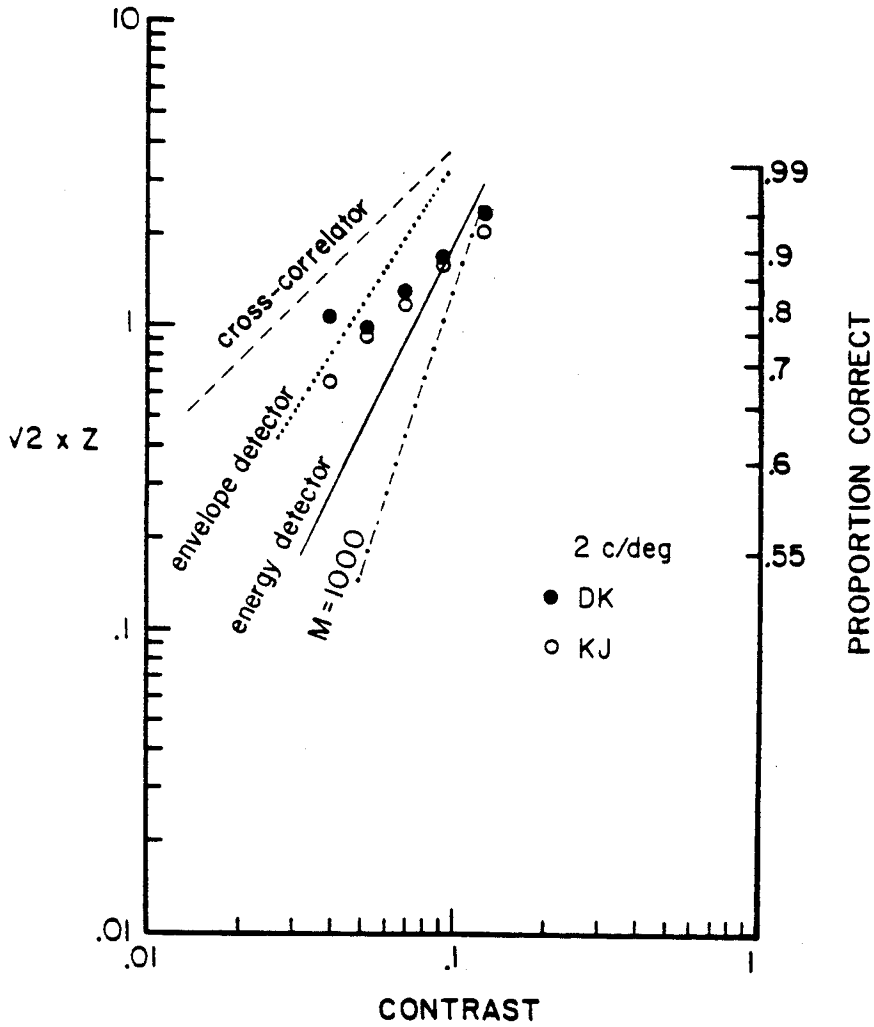

Fig. 4.4. The theoretical "psychometric functions" of Fig. 4.1 plotted with data from two observers, DK (filled circles) and KJ (open circles) (Exp. 2). The stimulus was a 2 c/deg grating, with space and time constants of .25° and 80 msec. The noise spectral density was 10^{-5} sec-deg. The slopes of the psychometric functions of the human observers are .85 and 1.1 for DK and KJ. These slopes are most closely matched by that of the cross-correlator which has a slope of 1. Human performance is better than an energy or a one-of-1000-orthogonal signals detector. Data for the other conditions are shown in Table 4.3.

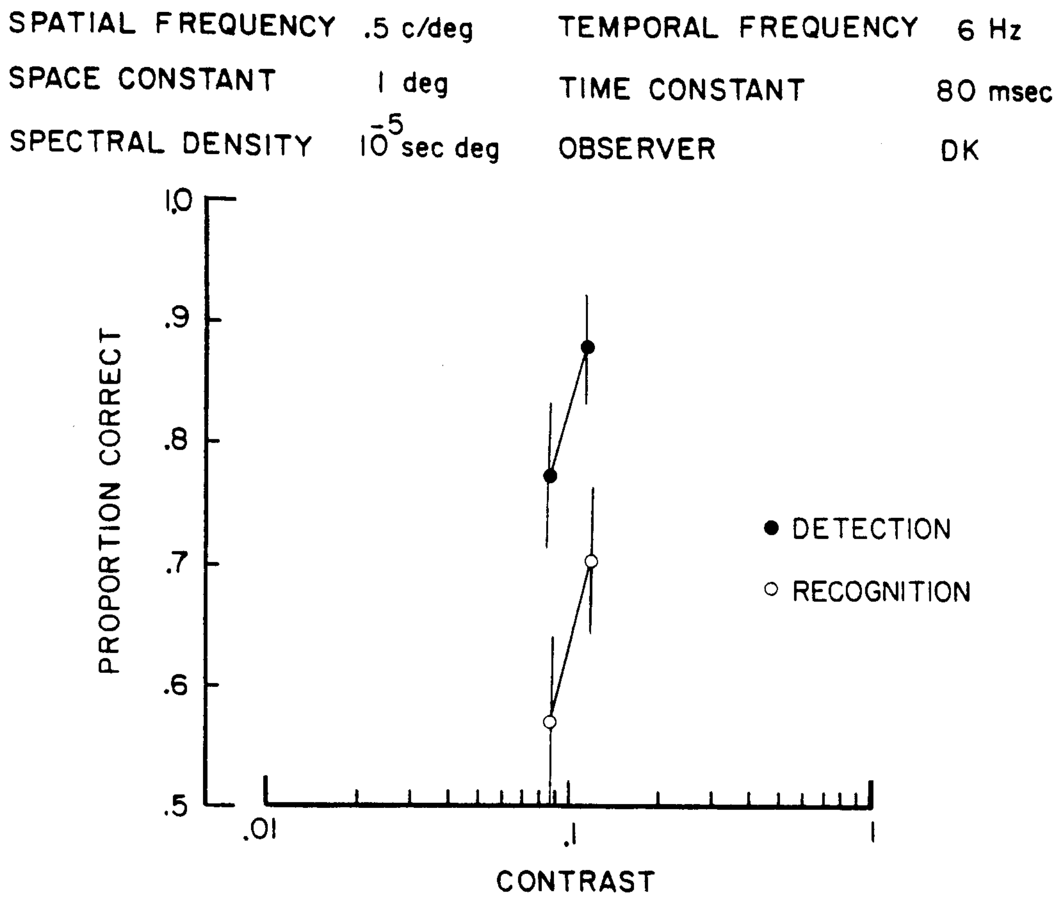

Fig. 4.5. Proportion correct as a function of contrast for detection (filled circles) and recognition (open circles) for a .5 c/deg grating drifting at 6 Hz in the presence of noise (Exp. 3). Observer DK. Data for the other conditions are shown in Table 4.4.

TABLE 4.4

Phase Recognition Data

Observer : DK

Spatial Frequency (c/deg)	Temporal Frequency (Hz)	Width (deg)	Noise Spectral Density (sec deg)	Contrast (%)	Signal Detected % correct ($100p_d$)	Phase Recognized % correct ($100p_r$)	Correlation Angle θ (deg)
.5	0	1.0	10^{-5}	8	86±5	89±4	107
				5	71±6	74±6	111
.5	6	1.0	10^{-5}	12	88±5	71±6	39
				9	78±6	57±7	19
.5	0	1.0	0	1.75	94±3	83±5	51
				1.31	72±6	58±7	28
2	0	.25	10^{-5}	10	86±5	88±5	100
				6	71±6	71±6	90

Observer : KJ

.5	0	1.0	10^{-5}	8	95±3	88±5	61
				5	69±6	73±6	122
.5	0	1.0	0	2	86±5	80±6	67
				1.3	81±5	63±7	31
2	0	.25	10^{-5}	10	89±4	84±5	70
				6	71±6	73±6	103

Observer : DR

.5	6	1.0	10^{-5}	14	97±3	61±7	12
				8	66±7	47±7	15

It is curious that the ability to recognize phase at detection threshold improves in the presence of noise as compared to no noise. Nachmias and Weber (1975) report several experimental results which describe the emergence of phase selectivity as contrast is raised. They showed that when a fundamental spatial frequency grating (3 c/deg) was above threshold, but its third harmonic was just detectable, the third harmonic's phase could be recognized as frequently as it could be detected. The third harmonic was either in cosine or anti-cosine phase relative to the fundamental. However, phase was not recognized as well when the fundamental was at threshold. Further, a suprathreshold fundamental could facilitate the detection of the third harmonic, whereas a high contrast third harmonic hindered the detection of the fundamental. Nachmias and Weber (1975) suggested that these results might indicate the presence of broad-band phase sensitive channels. This is consistent with the findings here. Phase recognition is good for exactly those stimuli whose detection resembles cross-correlation. The bandwidths of these stimuli are fairly broad, (≈ 1.7 octaves at half-height), and may match the bandwidths of available channels (see chapter 2).

SUMMARY

At the risk of some over-simplification, Table 4.5 summarizes the results of the three experiments according to partitioning described in RESULTS and DISCUSSION. The answer is shown below for the question posed above for each of the stimulus conditions.

The signal conditions can be partitioned into two categories which correspond to whether phase is very relevant to their detection or not. Thus phase is relatively important for the detection of the 2 c/deg and .5 c/deg gratings in noise. Any attempt to account for detection of these signals under these conditions needs to take into account phase. Of course, as pointed out above, this does not imply completely coherent detection. It was noted that about a factor of 2 in the efficiency for the 2 c/deg grating can be accounted for in terms of phase uncertainty.

Phase was not very important in the detection of the .5 c/deg grating when it was drifted in noise or when it was stationary in the absence of noise. This generalization should be qualified by the fact that some recognition can occur at detection threshold for both of these signals.

There is one final point to mention. It is not clear that a given result from one of the experiments is invariant over time or

TABLE 4.5

Summary of Results

Condition	Does Performance decline with phase uncertainty? ($C'<.1$?) Exp. 1	Psychometric function slope near one? Exp. 2	Is phase recognition relatively good compared to detection? ($\theta>90^{\circ}$?) Exp. 3	Phase-uncertain psychometric function slope steeper than phase-certain slope? (from data in Exp. 1)
2 c/deg in noise	yes	yes	yes	yes
.5 c/deg in noise	yes	yes	yes	yes
.5 c/deg drifting in noise	no	no	no	no
.5 c/deg no noise	no	no	no	no

over observers. With a suitable orienting task, or because of perceptual learning it might be possible to measure parameters which reflect a change in detection strategy. For example, when the observers in this study tried to recognize the phase of the .5 c/deg grating drifting at 6 Hz, they initially felt that the task was impossible. No data is available to say how fast they learned to do it. It might be hypothesized that observers might begin with a steep psychometric function which becomes progressively shallower as they learn to use the phase information. (The observers here only had a total of 400 trials.) It is also possible that a suitable orienting task could influence the slope of the psychometric function. For example, suppose in the third experiment, observers were required to recognize the width rather than the phase of a pair of gratings. We might expect to find an increase in the slope of the psychome function because observers now monitor more channels in an effort maximize correct responses on the recognition task.

Appendix 4.1

Ideal Psychometric Functions

Ideal observers for various signal detection tasks have been derived for numerous situations (e.g. Green and Swets, 1974). Most of these results have been derived for only one dimension. For many ideal detectors, existing theorems can be extended to several dimensions simply by defining noise spectral density over both positive and negative bandwidths. Then the signal-to-noise ratio is given by:

$$(s/n)^2 = (1/n^2) \int c^2(x,t)\,dx\,dt.$$

Here $c(x,t)$, the contrast function, is defined in terms of the luminance $L(x,t)$:

$$c(x,t) = (L(x,t) - L_o)/L_o,$$

where L_o is the average luminance over space and time. The noise spectral density n^2 is the variance of the contrast function of the noise divided by the two-sided spatial and temporal frequency bandwidths B_x and B_t. "Two-sided" means that the bandwidth is defined over positive and negative frequencies.

All the detectors used in this chapter are found in Green and Swets in some form. To translate their results, note that: E equals contrast energy (s^2) and N equals twice the noise spectral density (n^2) used here, because the noise spectral density in this chapter (and thesis) is defined over both positive and negative spatial frequencies. Bandwidth _W_, in Green and Swets, is equal to B/2 here, where _B_ is the two-sided bandwidth.

With these definitions in hand, the performance of the various ideal observers used in this chapter can be found. The performance of the cross-correlator is given by:

$$d' = s/n$$

where s^2 and n^2 are the contrast energy of the signal and noise spectral density of the noise mask.

A closed-form solution for the envelope detector involves the use of non-standard functions. Further, _d'_ can only be approximated under conditions where a quantity monotonic with likelihood ratio is normally distributed. Fortunately, Green and Swets (1974) plot the performance of the envelope detector for a two-alternative forced-choice procedure (see their Fig. 7.6).

Green and Swets (1974) also plot approximations for the one-of-M-orthogonal signal detectors in Fig. 7.6. However, these approximations are valid only over certain ranges (see Pelli, 1981). A better approximation for the performance for the one-of-M-orthogonal signals ideal is given by Pelli (1981). Psychometric functions were taken from Pelli's Fig. 5.3a.

The performance of the energy detector was derived in chapter 5 for the detection of a static noise signal in dynamic noise. An approximation for this energy detector is given by:

$$d' \approx \sqrt{\{8B_x/(\pi B_t s_x s_t)\}}\, c^2/N_o,$$

where B_x, B_t are the spatial and temporal two-sided bandwidths, and N_o is the static spectral density. The energy detector is not optimal for deterministic signals. Further, the above approximation breaks down when $B_x B_t s_x s_t < 10$. However, if rectangular spatial and temporal envelopes are assumed, then one can derive an approximate upper bound on performance. Let d'_x be given by:

$$d'_x = \frac{\sqrt{(B_x X/2)}\, m}{\{.5m^2 + m + 1\}^{1/2}}$$

where m is the spectral density signal-to-noise ratio (S_o/N_o) and X is the width. The denominator can be no smaller than 1, thus,

$$d_x' < \sqrt{(B_x X/2)}\, m.$$

As in Appendix 5.1, to take into account temporal summation, a factor $\sqrt{(B_t T)}$ is included,

$$d' < \sqrt{(B_x B_t XT/2)}\, m.$$

Let $c^2 = S_o B_x$, then

$$d' < \left\{\frac{B_t XT}{2B_x}\right\}^{1/2} \cdot \frac{c^2}{N_o}$$

For the 2 c/deg grating in noise, the stimulus conditions were $s_x = .25^o$, $s_t = .08$ sec and $n^2 = 10^{-5}$. Now let $B_t \approx 1.25/(\pi s_t)$ and $B_x \approx 1.25/(\pi s_x)$. Since N_o is equal to 10^{-3}, we have $d' < 198c^2$. The energy detector line shown in Figs. 4.1 and 4.4 plots d' as a function of $198c^2$.

REFERENCES

Atkinson, J. & Campbell, F.W. (1974). The effect of phase on the perception of compound gratings. Vision Res., 14, 159-162.

Burgess, A.E., Wagner, R.F., Jennings, R.J. & Barlow, H.B. (1981). Efficiency of human visual signal discrimination. Science, 214, 93-94.

Burr, D.C. (1980). Sensitivity to spatial phase. Vision Res., 20, 391-396.

Campbell, F.W. & Green, D.G. (1965). Optical and retinal factors affecting visual resolution. J. Physiol., 181, 576-593.

Campbell, F.W., Johnstone, J.R. & Ross, J. (1981). An explanation for the visibility of low frequency gratings. Vision Res., 21, 723-730.

Cohn, T.E., Thibos, L.N. & Kleinstein, R.N. (1974). Detectability of a luminance increment. J. Opt. Soc. Am., 64, 1321-1327.

Foley, J.M. & Legge, G.E. (1981). Contrast detection and near-threshold discrimination in human vision. Vision Res 21, 1041-1053.

Graham, N., Robson, J.G. & Nachmias, J. (1978). Grating summation in fovea and periphery. Vision Res., 18, 815-825.

Green, D.M. & Swets, J. (1974). Signal Detection Theory and Psychophysics. New York, John Wiley and Sons.

Horowitz, P. & Hill, W. (1980). The Art of Electronics. Cambridge, Cambridge University Press.

Jones, R.M. & Tulunay-Keesey, U. (1980). Phase selectivity of spatial frequency channels. J. Opt. Soc. Am., 70, 66-70.

Lasley, D.J. & Cohn, T.E. (1981). Why luminance discrimination may be better than detection. Vision Res., 21, 273-278.

Legge, G.E. (1976). Adaptation to a spatial impulse: Implications for Fourier transform models of visual processing. Vision Res., 16, 1407-1418.

Legge, G.E. &Foley, J.M. (1980). Contrast masking in human vision. J. Opt. Am., 70, 1458-1471.

Lindgren, B.W. (1976). Statistical Theory. 3rd ed. New York, Macmillan Pub. Co., Inc.

McCann, J.J., Savoy, R.L., Hall, J.A. Jr. & Scarpetti, J.J. (1974). Visibility of continuous luminance gradients. Vision Res., 14, 917-927.

Nachmias, J. (1972). Signal detection theory and its application to problems in vision. Handbook of Sensory Physiol., VII/4, Springer-Verlag.

Nachmias, J. & Sansbury, R.V. (1974). Grating contrast: Discrimination may be better than detection. Vision Res., 14, 1039-1042.

Nachmias, J. & Weber, A. (1975). Discrimination of simple and complex gratings. Vision Res., 15, 217-223.

Pelli, D. (1981). Effects of Visual Noise. Ph.D. Thesis. Cambridge University, England.

Peterson, W.W., Birdsall, T.G. & Fox, W.C. (1954). The theory of signal detectability. Trans. IRE Professional Group on Information Theory, PGIT-4, 171-212.

Piotrowski, L.N. (1981). The contributions of spatial-frequency amplitude and phase information for visual processing. 2nd International Conference on Visual Pschophysics and Medical Imaging, IEEE.

Roberts, T.A. (1963). Analysis and synthesis of linear and non-linear shift register generators. Internat. Telemetering Conf., 1, 390-399, IEE, Savoy Place, London.

Rubin, G.S. (1982). Suppression and summation in binocular pattern vision. Ph.D. Thesis. Department of Psychology. University of Minnesota.

Stromeyer, C.F. III, Klein, S., Dawson, B.S. & Spillman, L. (1982). Low spatial-frequency channels in human vision: Adaptation and masking. Vision Res., 22, 225-233.

Tanner, W.P. (1956). Theory of recognition. J. Acoust. Soc. Am., 28, 882-888.

Tanner, W.P. (1961). Physiological implications of psychophysical data. Ann. N.Y. Acad. Sci., 89, 752-765.

Tolhurst, D.J. & Dealy, R.S. (1975). The detection and identification of lines and edges. Vision Res., 15, 1367-1372.

Westheimer, G. (1978). Spatial phase sensitivity for sinusoidal grating targets. Vision Res., 18, 1073-1074.