Computational Vision
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Lecture 21: Texture

Initialize

Spell check off, plot options, etc..

```
Off[General::spell1];
```

```
SetOptions[ArrayPlot, ColorFunction → "GrayTones",
  DataReversed → True, Frame → False, AspectRatio → Automatic,
  Mesh → False, PixelConstrained → True, ImageSize → Small];
SetOptions[ListPlot, ImageSize → Small];
SetOptions[Plot, ImageSize → Small];
SetOptions[DensityPlot, ImageSize → Small, ColorFunction → GrayLevel];
nbinfo = NotebookInformation[EvaluationNotebook[]];
dir = ("FileName" /. nbinfo /. FrontEnd`FileName[d_List, nam_, ___] :> ToFileName[d]);
```

Histogram

```
histogram[image_, nbin_] := Module[{histx},
   Needs["Statistics`DataManipulation`"];
   histx = BinCounts[Flatten[image], {0, nbin - 1, 1}];
   Return[N[histx / Plus @@ histx]];
  ];
```

Entropy

```
entropy[probdist_] := Plus @@ (If[# == 0, 0, -# Log[2, #]] & /@ probdist)
```

Outline

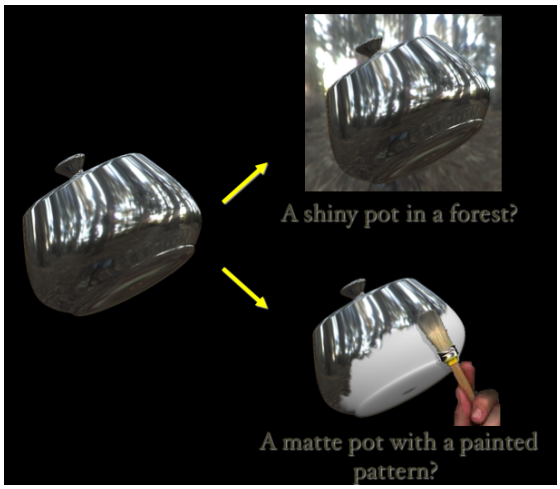
Last time

Surface material:

- Surface properties, color, transparency, etc..
- Reflectance & lightness constancy

Perception of shiny materials

Shiny or matte?



A major invariance problem.

Fleming et al. showed that the simple model of illumination with just one light source is not as effective as rendering in a realistic environment. But it isn't complexity per se, because white noise isn't good for conveying the underlying surface shininess. One of the main conclusions is that the presence of edges and bright points important, rather than recognizable reflected objects. http://journalofvision.org/3/5/3/article.aspx

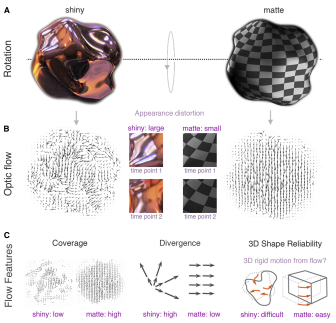
For background on HDR illumination probe measurements, see: http://www.pauldebevec.com/Probes/

And on the Uffizi probe see too:

http : // commons.wikimedia.org / wiki / Image : HDR_example _ – _exposure.jpeg

Motion and shininess

http://gandalf.psych.umn.edu/~kersten/kersten-lab/demos/MatteOrShiny.html



Cooperative computation

Transparences and structure from motion example

Today

Generative models for texture classes
The "generic" natural image model
Is human vision "tuned" to natural image statistics?

Generative models for texture

Understanding textures is important for material recognition, image segmentation, and understanding human visual performance, and the neural processing of visual information.

Types of textures

A useful distinction between types of textures is: deterministic and stochastic (Liu et al., 2010).

Look at texture samples in the databases below. Which ones appear more stochastic, and which more deterministic?

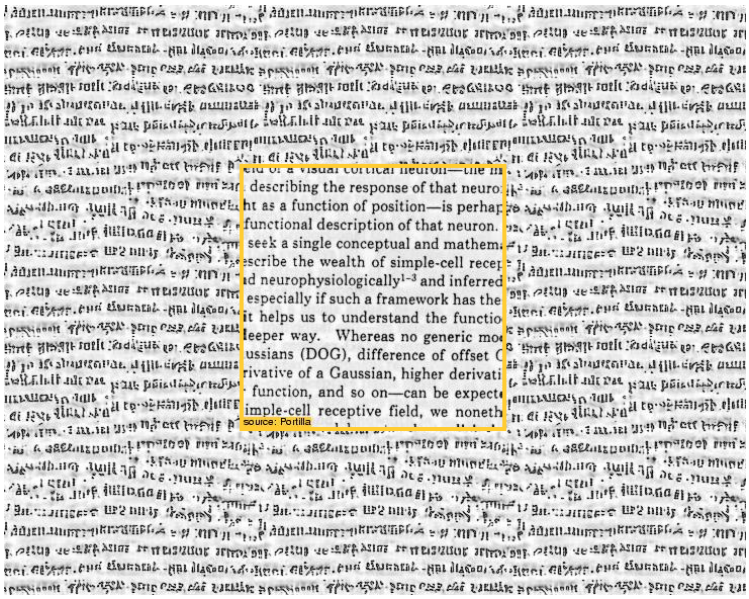
http://www.ux.uis.no/~tranden/brodatz.html
http://sipi.usc.edu/database/

Understanding how to *generate* textures is particularly important for building quantitative models of visual behavior and neural processing. For example, Freeman et al. (2013). A functional and perceptual signature of the second visual area in primates. Nature Neuroscience, 16(7), 974–981.

Text texture example from Javier Portilla and Eero Simoncelli

The demonstration below suggests that the phenomenon of “visual crowding” might have as its basis neural mechanisms that extract texture features in the periphery -- effectively summarizing visual information at a coarser level in the periphery than in the fovea. See: Freeman & Simoncelli, E. P. (2011). Metamers of the ventral stream. Nature Neuroscience, 14(9), 1195–1201.

http://www.cns.nyu.edu/~eero/texture/



We'll focus on stochastic textures because of their close relationship to many textures typically encountered in nature, with close connections to spatial filters in early vision.

Imagine an image ensemble consisting of all 256x256 images of "grass". This set is unimaginably large, yet there is a set of characteristic features that are common to all these images. Imagine we have an algorithm that from knowledge of these features can generate random image samples from this imaginary ensemble. One kind of algorithm takes a white noise image as input, and produces as output image samples that resemble grass. The white noise input behaves like fair roll of a high-dimensional die.

We show several methods for generating textures. And then we give an outline of one method for discovering the features from a small number of sample images.

There is a history of studies that seek to extract the essential features of a texture class (such as "grass" or "fur" or "all natural images"...) and then use these to build a texture synthesizer that produces new samples from the same texture class. A generative model provides a test of the extent to which the model has captured the essential statistics or features.

Texture synthesis using histogram matching

First-order intensity statistics. One of the simplest ways to do this would be to take what you've learned about intensity histograms, and then write a program that would produce new images by drawing pixel intensities from your model histogram, assuming each pixel is independent of the others. In other words, make i.i.d. random draws without consideration of any other pixel values.

Human vision shows sensitivity to first-order intensity statistics. For example, see Chubb et al., 2004.

Motoyoshi et al. (2007) showed a correlation between skew symmetry and the perception of material gloss. But simple statistics can only go so far, Kim et al. (2011).

► 1. Make a random image generator that draws samples from an intensity histogram measured from an natural image

Histogram matching between two images.

Suppose you have measured image statistics on a particular image, say a picture of grass. And now you want to force a white noise image to have the same statistics. Recall an earlier example of taking an natural image (a mountain lake) and forcing its intensity histogram to be flat ("whitening"). Suppose instead, we take a natural image, and want to force another image (perhaps a white noise image) to have the same histogram. This is called histogram matching. Histogram matching is useful when you want to have two images (or movies) look like they have the same lighting. But we can also ask whether we can use this method to make noise look more like texture from a specific texture class, like "grass".

Here's an example of where we generate a random image, and then constrain it to have the same first-order image statistics as another image--that of grass.

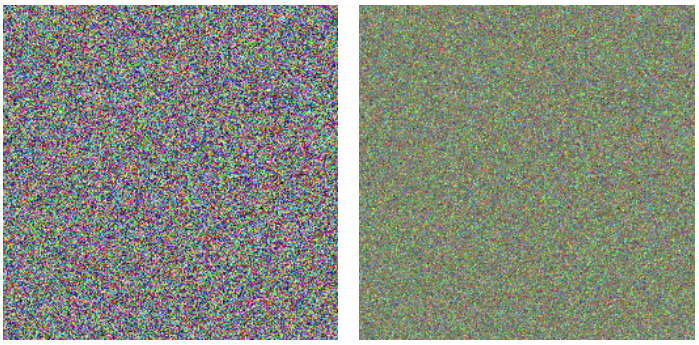
In[1]:=

In[2]:=
```
rimage = RandomImage[1, ImageDimensions[ ], ColorSpace → "RGB"];

mimage = HistogramTransform[rimage, ];

GraphicsRow[{rimage, ImageAdjust[mimage]}]
```

Out[4]=



2nd order statistics and random fractals

Second-order intensity statistics. Recall that one way to characterize the second-order statistics of a natural image is in terms of its auto-correlation function.

Also recall that the Fourier transform of the autocorrelation function is equal to the spatial power

spectrum of an image.

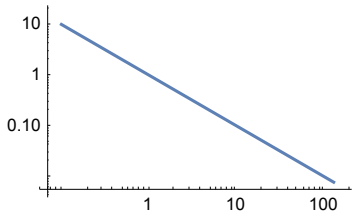
Natural images tend to have spatial frequency power spectra that fall off linearly with log spatial frequency (Simoncelli and Olshausen). When the slope of the fall-off is within a certain range, such images are called random fractals. The slope is related to the “fractal dimension”.

Random fractals can be characterized by the fractal dimension D (3<D<4) and amplitude spectrum, $1/(f_x^2 + f_y^2)$^(4-D). The amplitude spectrum is thus a straight line when plotted against frequency in log-log coordinates. The condition If[ ] is used to include a fudge term (1/(2)^(q)) to prevent blow up near zero in the Module[ ] routine below.

```
size = 256;
```

Random fractals have been suggested as good statistical models for the amplitude spectra natural images. Here is one way of generating them.

```
D1 = 3.5;
q = 4 - D1;
LogLogPlot[If[(i ≠ 0 || j ≠ 0), 1 / (i * i + 0 * 0) ^ (q), 1 / (2) ^ (q)],
 {i, .1, size / 2 - 1}, ImageSize → Small]
```



Here is a function to make a low-pass filter with fractal dimension D. (D, here should be between 3 and 4). Note that we first make the filter centered in the middle, and then adjust it so that it is symmetric with respect to the four corners.

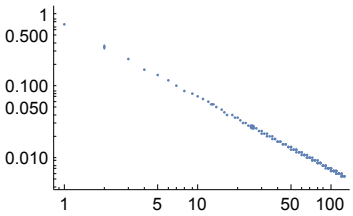
```
fractalfilter2[D_,size_] :=
Module[ {q,i,j,mat},
    q = 4 - D;
    mat = Table[If[(i != 0 || j!= 0),
        1.0/(i^2 + j^2)^q, 1.0/(2)^(q)],
    {i,-size/2,(size/2) - 1},{j,-size/2,(size/2) - 1}];
    Return[mat];
    ];
```

```
ft = Table[N[π (2 RandomReal[] - 1)], {i, 1, size}, {j, 1, size}];
ft = Fourier[ft];
randomphase = Arg[ft];
randomspectrum = Abs[ft];
```

```
fractalfilterarray = fractalfilter2[3.5, size];
ArrayPlot[fractalfilterarray^.2, Mesh → False]
```

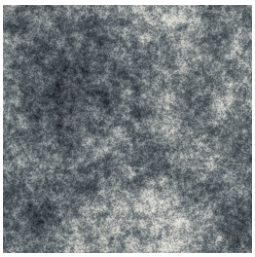
The exponent, .2, above is just for display purposes...it compresses the large values.

```
ListLogLogPlot[
 Table[RotateLeft[fractalfilterarray, {size / 2 + 1, size / 2 + 1}][[i, i]],
  {i, 1, size / 2}], ImageSize → Small]
```



Here is a random fractal image, with D = 3.5

```
ArrayPlot[Chop[
InverseFourier[RotateLeft[fractalfilterarray,{size/2,size/2}] randomspectrum Exp[I randomp
Mesh->False]
```



What does it look like?

Statistics from image pyramids

So far our samples are not capturing very interesting regularities.

To turn the noise sample into a texture that looks more like grass, or a random fractal to look more like a natural image, we need to force the synthesized patterns to match additional statistics. Samples from the fractal process modeled above are multi-variate Gaussian. But a limitation of Gaussian models is that they fail to capture phase structure, and in particular edges. Local edges can be thought of as regions where the the sine-phase fourier components all line up at zero frequency.

More generally, Heeger and Bergen (pdf) showed how to use image filter pyramids to generate novel textures from statistical "summaries" obtained from sample textures. They start off with a model of spatial filters that are selective for spatial frequency, orientation, and phase. The use of orientation filters captures oriented features of textures, and phase captures edges.

The filter model can be thought of as a model of the spatial filtering properties of V1 neurons. Then given a sample of a texture, measure the histograms for each of the filter outputs. The assumption is that these histograms summarize the essential features of the texture. Thus, given the histogram statistics, the goal of the algorithm is to produce new texture samples that have the same statistics but otherwise are random. One way to do this is to start of with a white or i.i.d. noise sample (i.i.d. meaning each pixel is independently drawn from an identical distribution, such as a uniform or gaussian distribution), and then iteratively adjust the noise sample to have the same histograms as learned from the original natural texture sample.

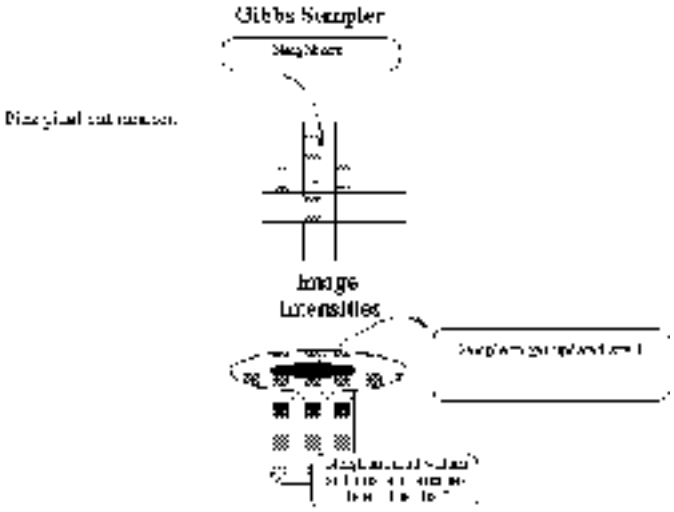
Texture synthesis using Markov Random Field models & Gibbs sampling

This section shows another way to synthesize textures using a model of the local conditional probabilities of intensities. In particular, we will show how to model piece-wise constant textures. The method it allows us, in theory, to generate samples from specified high-dimensional joint probability functions. By clamping (or fixing) nodes with known measurements, versions of this method can also be used to do inference.

Modeling textures using Markov Random Fields

Markov Random Fields (MRFs) have been used in computer vision and graphics for many years, and there is substantial body of literature. Some of the earliest papers are from Cross, G. R., & Jain, A. K. (1983) and Geman and Geman (1984). Rather than go into the theory, we can get an intuitive sense for MRFs by looking at how they can be used to generate textures.

Sampling from textures using local updates



The Gibbs Sampler (Geman and Geman, 1984)

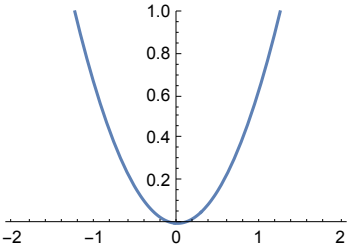
Set up image arrays and useful functions

In[51]:=
```
size = 64; T0 = 1.`; ngray = 16.`;
brown = N[Table[RandomReal[{1, ngray}], {i, 1, size}, {i, 1, size}]];
next[x_] := Mod[x, size] + 1;
previous[x_] := Mod[x - 2, size] + 1;
 Plus @@ Flatten[brown]
------------------------;
Length[Flatten[brown]]
```

Gaussian potential

In[56]:=
```
Clear[f]; (* Clear[f]; f[x_,n_]:=x^2;*)
f[x_, s_, n_] := N[(x / s) ^ 2];
s0 = 1.25; n0 = 2;
Plot[f[x, s0, n0], {x, -2, 2}, PlotRange → {0, 1}]
```

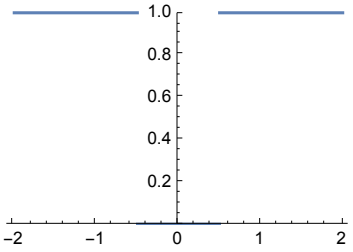
Out[59]=



Ising potential

In[60]:=
```
Clear[f]; (* Clear[f]; f[x_,n_]:=x^2;*)
f[x_, s_, n_] := If[Abs[x] < .5, 0, 1];
(*f[x_,s_,n_]:=N[(x/s)^2];*)
s0 = 1.; n0 = 5;
Plot[f[x, s0, n0], {x, -2, 2}, PlotRange → {0, 1}]
```

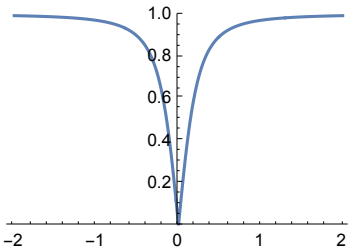
Out[63]=



Geman & Geman potential

In[64]:=
```
Clear[f]; (* Clear[f]; f[x_,n_]:=x^2;*)
f[x_, s_, n_] := N[Sqrt[Abs[x / s]^n / (1 + Abs[x / s]^n)]];
(*f[x_,s_,n_]:=N[(x/s)^2];*)
s0 = .25; n0 = 2;
Plot[f[x, s0, n0], {x, -2, 2}, PlotRange → {0, 1}]
```

Out[67]=



Define the potential function using nearest-neighbor pair-wise cliques

In[68]:=
```
Clear[gibbspotential, gibbsdraw, tr];
gibbspotential[x_, avg_, T_] :=
  N[Exp[-(f[x - avg[[1]], s0, n0] + f[x - avg[[2]], s0, n0] +
        f[x - avg[[3]], s0, n0] + f[x - avg[[4]], s0, n0]) / T]];
```

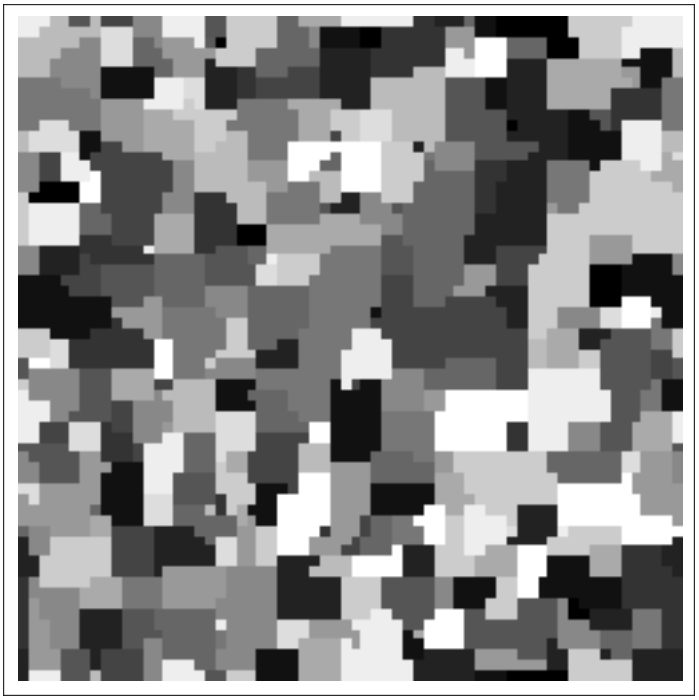
Define a function to draw a single pixel gray-level sample from a conditional distribution determined by pixels in neighborhood

In[70]:=
```
gibbsdraw[avg_, T_] :=
  Module[{}, temp = Table[gibbspotential[x + 1, avg, T], {x, 0, ngray - 1}];
   temp2 = FoldList[Plus, temp[[1]], temp];
   temp10 = Table[{temp2[[i]], i - 1}, {i, 1, Dimensions[temp2][[1]]}];
   tr = Interpolation[temp10, InterpolationOrder → 0];
   maxtemp = Max[temp2];
   mintemp = Min[temp2];
   ri = RandomReal[{mintemp, maxtemp}];
   x = Floor[tr[ri]];
   Return[{x, temp2}];];
```

"Drawing" a texture sample

In[71]:=
```
gd = ArrayPlot[brown, Mesh → False, PlotRange → {1, ngray}];
Dynamic[gd]
```

Out[72]=



In[73]:=
```
For[iter = 1, iter ≤ 10, iter++, T = 0.25`;
 For[j1 = 1, j1 ≤ size size, j1++, {i, j} = RandomInteger[{1, size}, 2];
  avg = {brown[[next[i], j]], brown[[i, next[j]]],
    brown[[i, previous[j]]], brown[[previous[i], j]]};
  brown[[i, j]] = gibbsdraw[avg, T][[1]];];
 gd = ArrayPlot[brown, Mesh → False, PlotRange → {1, ngray}]]
```

Was it a true sample? Drawing true samples means that we have to allow sufficient iterations so that we end up with images whose frequency corresponds to the model. How long is long enough?

Finding modes

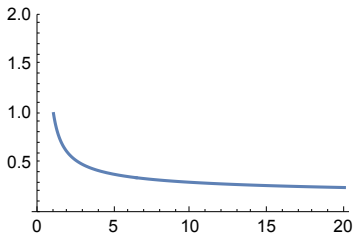
A texture model can be used as a Bayesian prior that captures the regularities in the class of pictures. The texture model can be used similarly to the "smoothness" priors we encountered before. For example, if a natural image is corrupted by white noise, one can construct a posterior distribution that encourages the "cleaned up image" to be close to the original but also closer to the prior. To compute the most probable original image can use Gibbs sampling on the prior, but now we would like to find the mode of the distribution, not just draw samples.

To illustrate, let's just consider the texture prior. Suppose you want to find the texture which is the most probable. There may not be just one, but there are tricks to try to find a texture sample that is the most probable, or at least more probable than any preceding ones in the iterative process.

Simulated annealing provides one such method. Simulated annealing can be used in many other domains that require finding solutions to high-dimensional optimization problems, including ones in vision. A classic example in neural networks is the Boltzmann machine.

Define annealing schedule

```
anneal[iter_, T0_, a_] := T0 * (1 / a) / (1 / a + Log[iter]);
Plot[anneal[iter, T0, 1], {iter, 1, 20}, PlotRange → {0, 2}]
```



"Drawing" a texture sample with annealing

```
gd2 = ArrayPlot[brown, Mesh → False, PlotRange → {1, ngray}];
Dynamic[gd2]
```

gd2

```
For[iter = 1, iter ≤ 10, iter++, T = anneal[iter, T0, 1];
  For[j1 = 1, j1 ≤ size size, j1++, {i, j} = RandomInteger[{1, size}, 2];
   avg = {brown[[next[i], j]],
     brown[[i, next[j]]], brown[[i, previous[j]]], brown[[previous[i], j]]};
   brown[[i, j]] = gibbsdraw[avg, T][[1]];];
  gd2 = ArrayPlot[brown, Mesh → False, PlotRange → {1, ngray}]];
```

Machine learning of distributions on textures

A fundamental problem in learning image statistics that are sufficient for generalization and random synthesis is that images have enormously high dimensionality compared with the size of a reasonable database. So when trying to construct a model, one runs the risk of "over-fitting". When trying to estimate conditional distributions, there may never be enough data to fill the histograms.

These are some of the problems of the "curse of dimensionality". One method to deal with characterizing high dimensional textures is to seek out probability distributions that have the same statistics (i.e. a small finite set of statistical features) as those measured from an available database (e.g. "1000 pictures of grass"), but are minimally constraining in other dimensions.

Suppose one has a collection of probability distributions that all have the same statistics. At one extreme, the original database itself defines a distribution--a random draw is just a pick of one of the pictures. But unless this distribution is impossible large, it has no "creativity" and leaves out a huge set of grass images that are not in the database. At the other extreme, is the distribution which has the specified statistics but has maximum entropy (Cover and Thomas, 1991). The connection between maximum entropy and the principle of symmetry in probability theory was described in the *Mathematica* notes on probability theory.

Minimax entropy learning: Zhu et al.

This section provides a brief outline of work by Zhu, S. C., Wu, Y., & Mumford, D. (1997). Minimax Entropy Principle and Its Applications to Texture Modeling. Neural Computation, 9(8), 1627-1660. It provides a theoretical framework for learning "flat" models of images--i.e. texture models. Recent work seeks to extend this theory to hierarchical models of image structure (Yuille, 2011).

See the References for other work on texture learning and modeling.

Maximum entropy to determine $p_M(I)$ which matches the measured statistics, but is "least committal"

Suppose we have a set of filters ϕ_i. An example would be a simple difference filter such as a discrete approximation to a ∇^2operator, which we saw produces histograms from natural images with high kurtosis. An example would be the set of filters from a pyramid decomposition:

```
{ϕi (I) : i = 1, ..., N}
```

Given a collection of image samples I, we measure the values of the filter outputs, i.e. texture statistics, ψ_i. Think of these statistics as the histogram values themselves, i.e. for each filter we would get n histogram probability estimates, where n is the number of bins.

A good model of the texture p_M would have the same statistics as the true underlying model model, p(I). (We don't know p(I), but we have samples from it--the ones we used to calculate the statistics.)

$$\sum_{I} p_M(I)\,\phi_i(I) = \psi_i, \quad \text{for } i = 1, \ldots, N$$

But there is an enormous family of possible probability distributions $\{p_M(I)\}_M$ that could all have the same statistics. If we want a texture model that has maximum "creativity", we can model this constraint by looking for the distribution that has the highest entropy, while constrained to have the required statistics.

Zhu et al.'s method built on a a standard method in information theory (Cover and Thomas, 1991) to obtain the maximum entropy distribution for a given set of measured statistics. The idea was to "learn" the form of the potentials λ_i (as in the Ising potential assumed above).

$$p_M(I) = \frac{1}{Z[\lambda]} \exp\left\{-\sum_{i=1}^{N} \lambda_i\,\phi_i(I)\right\}$$

Minimum entropy to determine statistics/features

But what features (i.e. filters) are the most important? How do we know what to include--e.g. all of the histograms, one for each spatial filter, over a pyramid? Or can we get by with less. Or perhaps, we have an even larger potential set of filters, including ones from some other filtering scheme.

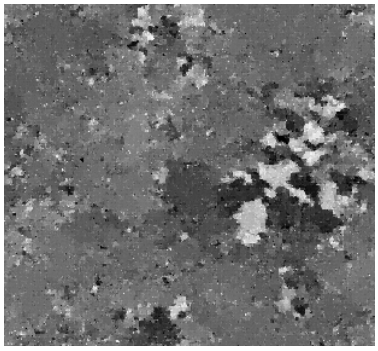
The number of filters needed will depend on the texture and the initial choice of feature set. Suppose one has a filter set modeled after V1 spatial filters. Some filters may be much more important than others in capturing the essential statistics. Assume that p(I) is the true model that has all of the essential statistics. This could be really complex, and we don't know for sure what filters to include. So Zhu et al.'s idea was to do something analogous to a Taylor series expansion, and order filters so that as one added more filters to p_M, it gets us closer to the true distribution $p(I)$. But at some point, adding more filters doesn't move us closer to the true distribution.

To do this, one needs a measure of "distance" between two distributions. We've already learned about d' in a completely different context. A more general measure is Kullbach-Leibler divergence (wiki): D(p(I) | p_M(I)). Zhu et al. showed that by choosing filters that *minimize the entropy* of p_M(I), they could move the distribution in the direction towards p(I).

$$D(p(I) \mid p_M(I)) = \text{entropy}(p_M(I)) - \text{entropy}(p(I))$$

So while you don't know p(I), you can at least try to move your texture distribution towards it.

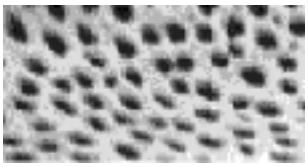
Here is a sample from a generic prior. Generic means they trained the model on samples of natural images without regard for particular texture classes.



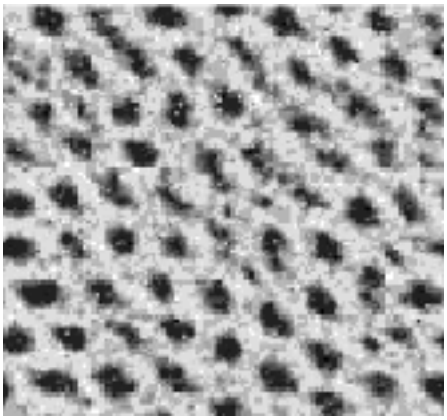
Here is a sample from a class-specific prior. Here the distribution was learned based on samples of “cheetah” fur.

Song Chun Zhu, Zhu & Mumford, IEEE PAMI, Zhu, Wu, Mumford, 1997

Original texture



Synthesized sample using Gibbs sampler



Nonparametric sampling

While theoretically well-grounded, the above approach can be difficult to implement efficiently. A more practical approach is suggested by Claude Shannon's approach to synthesizing English (Shannon, 1948; 1951). Imagine you have a large corpus of english language. You randomly pick a letter and write it down. To get the next letter you start a random search in your corpus for that letter, and when you find one, write down the letter after that, and so on. But you can imagine taking into account higher-order probabilities by searching for two- or N-letter combinations in your corpus, and writing down the letter immediately following those two. Of course, if N gets too big, even a large corpus won’t be big enough. So another alternative is to synthesize at the word, rather than letter level, using the same strategy.

Efros' application to textures is analogous to Shannon's method for generating text. Instead of first estimating the local MRF distributions (conditional value of a pixel given its neighbors), imagine starting off with a small "seed" (e.g. 3x3 pixels), and then querying the original sample image (playing the role of the corpus) to find similar neighborhoods to constrain how to make the draws to add pixels, one by one, to the neighborhood of the seed. See :

http://graphics.cs.cmu.edu/people/efros/research/EfrosLeung.html

Deep dreams?

So how can one generate samples that have an even richer structure? One method is to first train a deep neural convolutional network to classify object categories in natural images using backpropagation to adjust the weights. After learning, one can use backpropagation to perturb the original image to maximize the output of a particular high-level "neuron", e.g. one that prefers images of say birds. See: deep dream generator works: http://deepdreamgenerator.com

We will learn more about this later when we study object recognition, and methods to learn to recognize.

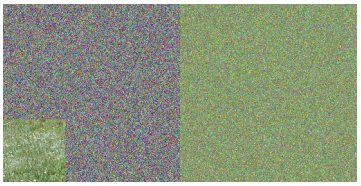
Appendix

Here's an adaptation from one of the *Mathematica* demonstrations. It shows alternative code for doing histogram matching

```
img1 =  ;
```

```
img2 = RandomImage[NormalDistribution[.5, .2], {256, 256}, ColorSpace → "RGB"];
```

```
𝒟1s = Map[HistogramDistribution[Flatten[#], 256] &,
    ImageData[img1, Interleaving → False]];
𝒟2s = Map[HistogramDistribution[Flatten[#], 256] &,
    ImageData[img2, Interleaving → False]];
{Tred, Tgreen, Tblue} = MapThread[FunctionInterpolation[
      InverseCDF[#1, CDF[#2, x]], {x, 0, 1}, AccuracyGoal → 1] &, {𝒟1s, 𝒟2s}];
res = ImageApply[{Tred[#[[1]]], Tgreen[#[[2]]], Tblue[#[[3]]]} &, img2];
ImageAssemble[{ImageCompose[img2,
    ImageResize[img1, Scaled[.35]], {Left, Bottom}, {Left, Bottom}], res}]
```



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