Initialize

Off[General::spell1];
<< Graphics`

Outline

Last time

• Early motion measurement—types of models
  correlation
  gradient
  feature tracking
• Functional goals of motion measurements
• Optic flow
  Cost function (or energy) descent model
  A posteriori and a priori constraints
  Gradient descent algorithms
  Computer vs. human vision and optic flow
    — area vs. contour

Today

■ Motion phenomena
Neither the area-based nor the contour-based algorithms we've seen can account for the range of human motion phenomena or psychophysical data that we now have.
Look at human motion perception

■ Local measurements
Representing motion, Orientation in space-time
  Fourier representation and sampling
  Optic flow, the gradient constraint, aperture problem
Neural systems solutions to the problem of motion measurement.
  Space-time oriented receptive fields

■ Global integration
Sketch a Bayesian formulation—the integrating uncertain local measurements with the right priors can be used to model a variety of human motion results.

Human motion perception

Demo: area-based vs. contour-based models
Last time we asked: Are the representation, constraints, and algorithm a good model of human motion perception?
The answer seems to be “no”. The representation of the input is probably wrong. Human observers seem to give more weight to contour movement than to intensity flow. Human perception of the sequence illustrated below differs from "area-based" models of optic flow such as the above Horn and Schunck algorithm. The two curves below would give a maximum correlation at zero—hence zero predicted velocity. Human observers see the contour move from left to right—because the contours are stronger features than the gray-levels. However we will see in Adelson’s missing fundamental illusion that the story is not as simple as a mere "tracking of edges"—and we will return to spatial frequency channels to account for the human visual system's motion measurements.
Let's make a 2D gray-level picture displayed with `ListDensityPlot` to experience the Mach bands for ourselves. `PlotRange` allows us to scale the brightness.

Aperture effects

- Circular aperture

```math
\text{niter} = 8; \text{width} = 64; \\
\theta_1 = \pi/4.; \text{contrast}_1 = 0.5; \\
\text{freq}_1 = 4.; \text{period}_1 = 1/\text{freq}_1; \\
\text{stepx}_1 = \cos(\theta_1) \times (\text{period}_1/\text{niter}); \text{stepy}_1 = \sin(\theta_1) \times (\text{period}_1/\text{niter}); \\
\text{grating}[x, y, \text{freq}, \theta_1] := \cos[(2. \pi \text{freq}) \times (\cos(\theta_1) x + \sin(\theta_1) y)];
```

- Square aperture

```math
\text{For}[i = 1, i < \text{niter} + 1, i++, \\
\text{DensityPlot}[\text{If}[(x-0.5)^2+(y-0.5)^2 < 0.3^2, \text{grating}[x+i \times \text{stepx}_1, y+i \times \text{stepy}_1, \text{freq}_1, \theta_1], 0], \\
\{x, 0, 1\}, \{y, 0, 1\}, \\
\text{Mesh} \rightarrow \text{False}, \text{Frame} \rightarrow \text{None}, \text{PlotRange} \rightarrow (-2, 2), \text{PlotPoints} \rightarrow \text{width}];
```

What do you see at the vertical boundaries? The horizontal boundaries?
Rectangular horizontal aperture

\[
\text{For}[i=1,i<niter+1,i++,
\text{DensityPlot}[\text{grating}[x+i*\text{stepx1},y+i*\text{stepy1},\text{freq1},\text{theta1}],
\{x,0,1\},\{y,0,.25\},
\text{Mesh}\to\text{False},\text{Frame}\to\text{None},\text{PlotRange}\to\{-2,2\},\text{PlotPoints}\to\text{width},
\text{AspectRatio}\to\text{Automatic}];
]\]

Rectangular vertical aperture

\[
\text{For}[i=1,i<niter+1,i++,
\text{DensityPlot}[\text{grating}[x+i*\text{stepx1},y+i*\text{stepy1},\text{freq1},\text{theta1}],
\{x,0,.25\},\{y,0,1\},
\text{Mesh}\to\text{False},\text{Frame}\to\text{None},\text{PlotRange}\to\{-2,2\},\text{PlotPoints}\to\text{width},
\text{AspectRatio}\to\text{Automatic}];
]\]

Adelson’s missing fundamental motion illusion

We first make a square-wave grating.

\[
\text{realsquare}[x_,y_,\text{phase}_] := \text{Sign}[\text{Sin}[x + \text{phase}]];
\]

And make a four-frame movie in which the grating gets progressively shifted LEFT in steps of \(90\) degrees.

\[
\text{For}[i=0,i<4, i++,
\text{DensityPlot}[\text{realsquare}[x,y,i \pi/2],
\{x,0,14\},\{y,0,1\}, \text{Frame}\to\text{False},
\text{Mesh}\to\text{False},\text{PlotRange}\to\{-4,4\},\text{PlotPoints}\to 60,\text{Axes}\to\text{None}];
]\]

Now subtract out the fundamental frequency from the square wave

\[\text{realmissingfundamental}[x\_\_\text{y\_\_\_\_\_phase\_\_\_\_}] := \text{realsquare}[x\_\_\_\text{y\_\_\_\_\_phase\_\_\_\_}] - \left(\frac{4}{\pi}\right) \text{Sin}[x + \text{phase}]\]

And make another four-frame movie in which the missing fundamental grating gets progressively shifted LEFT in steps of \(90\) degrees.

It is well-known that a low contrast square wave with a missing fundamental similar to the same square wave (there is a pitch analogy in audition). One reason is that we are more sensitive to sharp than gradual changes in intensity. If you look at the luminance profile of the grating to the left, as before. But it doesn’t. Surprisingly, the missing fundamental wave appears to move to the right!

\[
\text{For}[i=0,j<4, i++,
\text{DensityPlot}[\text{realmissingfundamental}[x\_\_\text{y\_\_\_\_\_phase\_\_\_\_}],
(x,0,14),(y,0,1), \text{Frame}\to\text{False},
\text{Mesh}\to\text{False},\text{PlotPoints} \to 60,\text{Axes}\to\text{None},\text{PlotRange}\to\{-4,4\}];
]\]

Play the above movie. It typically appears to be moving to the right. You can generate movies with different contrasts by adjusting the PlotRange parameters.

In fact the missing fundamental frequency moves towards the left as you can see by playing the movie below.

\[
\text{For}[i=0,j<4, i++,
\text{Plot}[\text{Sin}[x + i \pi/2],\{x,0,14\},
\text{PlotPoints} \to 60,\text{Axes}\to\text{None}];
]\]
What in the stimulus does move to the right?

Why might this be? Probably the best explanation comes from looking at the dominant frequency component in the pattern, which is the 3rd harmonic. It turns out that the third harmonic is jumping in 1/4 cycle steps to the right, even though the pattern as a whole is jumping in 1/4 cycle steps (relative to the missing fundamental) to the left, as shown in the figure below:

Make a movie with Plot[] that shows the third harmonic. Which way does it move?

And here is the movie with just the third harmonic. Which way does it move?
The main conclusion drawn from this demonstration is that human motion measurement mechanisms are tuned to spatial frequency.

How can the inferred biological mechanisms be pieced together to compute optic flow? We can construct the following rough outline. (For a recent algorithm for optic flow based on biologically plausible spatiotemporal filters see Heeger, 1987). Assume we have, at each spatial location, a collection of filters tuned to various orientations ($\theta$) and speeds ($s$) over a local region. (Already we run into problems with this simple interpretation, because many V1 cells are known to be tuned to spatial and temporal frequency in such a way that the spatio-temporal filter is the product of the space and time filters. This means that there is a favored temporal frequency that is the same across spatial frequencies, so the filter will be tuned to different speeds depending on the spatial frequency).

In this scheme, the optic flow measurements are distributed across the units, so if we wanted to read off the velocity from the pattern of activity, we would need some additional processing. For example, the optic flow components could be represented by the "centers of mass" across the distributed activity. Because these measurements are local, we still have the aperture problem. We will look at possible biological solutions to this problem in the next lecture.

**Motion plaids**

Two overlapping (additive transparent) sinusoids at different orientations and moving in different directions are, under certain conditions seen as a single pattern moving with a velocity consistent with an intersection of constraints. Under other conditions, the two individual component motions are seen.
Orientation in space-time

Representation of motion

**Mathematica demo**

```mathematica
size = 32; x0 = 4; y0 = 4; pw = 12; xoffset = 1;
A1 = Table[Random[], {size}, {size}]; (*A2 = A1;*)
A2 = Table[Random[], {size}, {size}];
A2[[Range[y0, y0 + pw], Range[x0, x0 + pw]]] =
A1[[Range[y0, y0 + pw], Range[x0 + i, x0 + pw + i]]];
x0 = Join[x0, {A2[[8]]}];
ListDensityPlot[A1, Mesh -> False, Frame -> False];
ListDensityPlot[A2, Mesh -> False];
```

**x-y-t space**

10.3 A motion sequence is a series of images measured over time. (A) The motion sequence of images can be grouped into a three-dimensional volume of data. (B) Cross sections of the volume show the spatial pattern at a moment in time. (C) Time (t) may be plotted against one dimension (x) of space. When the spatial pattern is one-dimensional, the (t, x) cross-section provides a complete representation of the stimulus sequence.
Neurophysiological filters

Space-time filters for detecting orientation in space-time

1. Space-time filters for detecting orientation in space-time

   \[ \frac{\partial L}{\partial x} + \frac{\partial L}{\partial y} + \frac{\partial L}{\partial t} = 0 \]
   \[ \frac{\partial L}{\partial x} + \frac{\partial L}{\partial t} = 0 \]

   Image \( I(x,y,t) \rightarrow \) blurred in space and smeared in time, \( g(x,y,t) \)

   \[ v_x \frac{\partial g}{\partial x} + \frac{\partial g}{\partial t} = 0 \]

From Wandell, "Foundations of Vision"
Bayesian model for integrating local motion measurements

Global integration.

Recall Lorenceau & Shiffrar's demo

Intersection of constraints revisited

Grating plaid sometimes seen as coherent, other times as two overlapping transparent gratings moving separately.
Weiss & Adelson's Bayes model for integration

- **Probabilistic interpretation of intersection of constraints**

![Diagram](image)

- **Probabilistic interpretation with noisy measurements**

![Diagram](image)

- **Generalize to other types of motion stimuli**

  Requirements for generalization:

  Base likelihoods on actual image data
  spatiotemporal measurements
  Include "2D" features
  E.g. corners
  Rigid rotations, non-rigid deformations
  Stage 1: local likelihoods
  Stage 2: Bayesian combination
  - Prior
  - slowness -- wagon wheel example, quartet example
  - smoothness - e.g. translating rigid circle
Overview of Weiss & Adelson theory

Dense to sparse:
\[ v(r) = \Phi(r)\theta \]
\[ v_x(x, y) = \sum_{i=1}^{N/2} \theta_i G(x - x_i, y - y_i) \]
\[ v_y(x, y) = \sum_{i=1+N/2}^{N} \theta_i G(x - x_i, y - y_i) \]

Likelihood:
\[ L(v) \propto e^{-\sum_r W(r)(I_i v_i + I_j v_j + I_l)^2 / 2 \sigma^2} \]
\[ L_r(v) \rightarrow p(I|\theta) \propto \prod_r L_r(\theta) \]

Prior:
\[ P(V) \propto e^{-\sum_r (Dv)^2(r)(Dv)(r)/2} \]
\[ P(V) \rightarrow P(\theta) \]

Posterior:
\[ P(\theta | I) \propto P(I | \theta)P(\theta) \]

Log posterior is quadratic in \( \theta \), \( \rightarrow \) linear estimator for \( \theta \)

Weiss & Adelson, 1998

Tests of theory

Rhombus experiment

Aperture effects
From Weiss and Adelson, 1998. Type I and II plaids. (Yo and Wilson, 1992)

Appendices

References


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