HW4: Answer Key

1) For the simulation in part 1 (FOV = 64 x 64 mm; matrix size = 64 x 64) what is the in-plane resolution?

Resolution = FOV ÷ matrix size = [64 64]./[64 64] = 1 x 1 mm

What is kmax?

k_{max} is half the matrix size (since k = 0 is in the center of the matrix) times delta-k:

delta-k = 1/FOV = 1/.064 m = 15.6 m^{-1} ; kmax = 32 * 15.6 m^{-1} = 499 m^{-1}

2) Although this isn't part of the simulation, if the slice-select gradient is 30 mT/m (3 G/cm) and the pulse bandwidth is 1 kHz, what is the slice thickness?

slice thickness = pulse BW ÷ Gradient [converted to Hz] = BW / (γG) = 2 kHz / (42.58 MHz/T * 32 mT/m)

= 2 x 10^3 Hz / (42.58 x 10^6 Hz/T * 32 x 10^{-3} T/m = 7.8 x 10^{-4} m = 1.5 mm

3) If the read-out gradient strength is 37 mT/m, what dwell time (time to acquire each next data point) would you need to generate a k-space step size of 15.6 m-1?

$\Delta k = \gamma G \Delta t$

$\Delta t = \Delta k/(\gamma G)$ = 15.6 m^{-1} / (42.58 MHz/T * 37 mT/m) = 15.6 m^{-1}/42.58x10^6Hz/T*37x10^{-3} T/m

= 1.0 x 10^{-5} Hz^{-1} = 1.0 x 10^{-5} s = 10 x 10^{-6} s = 10 μs

4) What is the sampling (digitization) bandwidth associated with the above dwell time?

sampling BW = 1 / dwell time = 1 / 10 x 10^{-6} s = 100,000 s^{-1} = 100,000 Hz

5) What FOV corresponds to the k-space step size given in (3)?

FOV = 1/Δk = 1 / 15.6 m^{-1} = 0.064 m = 64 mm [Note that this matches the information given in (1).]

6) If you're acquiring 64 data-points, with the step-size calculated in part (4), what are the minimum and maximum k-values you read out?

The data matrix is centered in k-space on k=0 [m^{-1}]. With a total matrix size of 64 x 64, there are 32 points to either side of 0, each separated by 15.6 m^{-1}, so the minimum k-space value is -32*15.6 m^{-1} = -499 m^{-1}, and k_{max} = 32*15.6 m^{-1} = 499 m^{-1}.

Although, to be perfectly accurate, the 33rd matrix element is actually 0, so there are 32 to the left and only 31 to the right … but we can infer that "missing" 32nd positive k value from the acquired negative k value, so functionally we have k-space spanning ±500 m^{-1}.

7) Given k_{min} and k_{max} from (6), what is your resolution?

resolution = 1 / FOV_k = 1 / (k_{max} – k_{min}) = 1/(998 m^{-1}) = 0.001 m = 1 mm

Conveniently, this matches the answer to question #1.

8) And, finally ... 2 questions ... a) If you want to increase the resolution, keeping the FOV the same, what do you do?

Keeping the FOV the same means keeping Δk the same, so $\gamma G \Delta t$ can't change. Increasing the resolution means increasing k_{max}, so you can just keep sampling longer … or you can increase G while decreasing Δt (proportionately) so you're sampling faster and covering more of k-space in the same amount of time.

b) if you want to increase the resolution, keeping the image matrix size the same (acquire same # of data points), what do you do to the strength of the read-out gradient?

Increasing the resolution while keeping the matrix size the same means you have to cover more of k-space in the same number of steps. To do this, you need to take bigger k-space steps (increasing Δk), which means decreasing the FOV. Makes sense: higher resolution, same matrix size … FOV must be smaller. To increase Δk, you need to increase the strength of the read-out gradient or decrease the sampling bandwidth (increase Δt).

In lab we played with this idea by setting up a pulse sequence so that G was already as large as it could be. Then, to decrease the FOV (increase Δk), we needed to have a lower bandwidth (larger Δt).