HW2: Answer Key

Question 1: Which sample is farther off resonance

The blue one. You can see this in the bottom plots. On the left (time domain), the blue line is oscillating faster. It dies out so quickly, it's hard to see, but it's going up and down twice as fast as the black line. That means that, relative to the reference frame, it's spinning faster than the black line. On the right (frequency domain), the center of the peak is farther away from 0 Hz. We're always plotting relative frequencies, so we don't know what the actual frequencies are, but we can tell that the blue signal is, on average, more Hz away from our reference.

Question 2: Can we tell what our reference frequency is?

Not from any of the plots here! It just gets subtracted out, and everything is quantified **relative** to the reference (which is therefore 0 Hz).

Question 3: Does the phase map on the left belong to the blue or black line?

Blue! Phase maps show the relative phase for the (average of all the) sines in every pixel in your image. Since phase is like the big hand on the clock (you can only tell the minute, not the hour), every time an off-resonance spin gets 2-pi ahead of the rotating frame, it looks like it's back at 0. So all those discontinuities in the phase map, which make stripes, are showing locations where spins have once again made an extra lap, relative to an (imaginary) on-resonance spin. So more stripes = more off-resonance.

Question 4: What would the signal look like in the frequency domain if there was not a single spin off resonance?

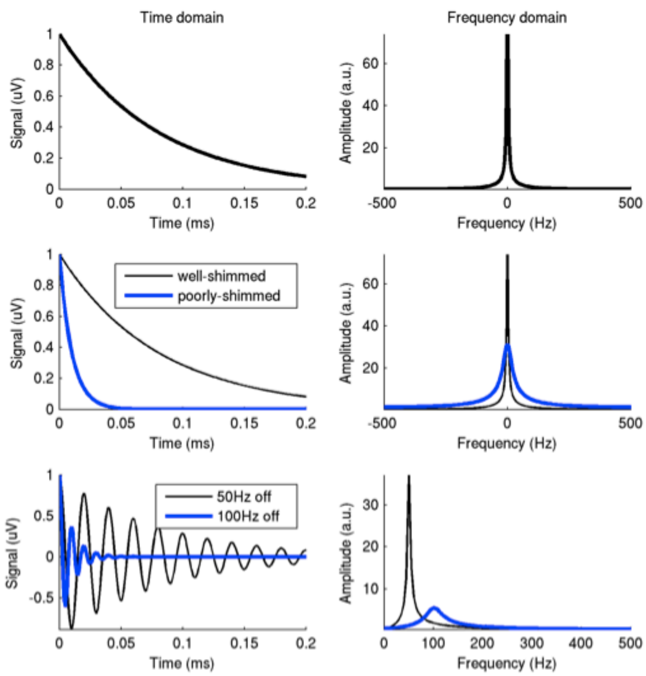
In the frequency domain, the plot would be a single spike … at 0, if it's exactly on your reference frequency, or off of 0 if it's off-resonance but still perfectly shimmed.

In the frequency domain, it would never decay … so the signal would just sit there and oscillate.

The line representing resonant frequency would be very (infinitely?) skinny. If the non-decaying isochromat is at our reference frequency, it will be on f=0. It is, however, possible that the non-decaying signal would be off-resonance, in which case we would find it at a different location (but still very skinny and very tall) on the frequency axis.

To invoke a few more rules about Fourier transforms:

- the amplitude of the Fourier transform at zero frequency would be equal to the mean of the time-domain signal (if you normalize by the number of points in the time-series, which isn't shown below …)
- the integral of the Fourier transform is related to the variance of the time-domain signal … which is zero, so the integral of our infinitely-skinny line is 0 … since it's peak (amplitude) is equal to the time-domain mean, it's width must be 0 for this to hold.

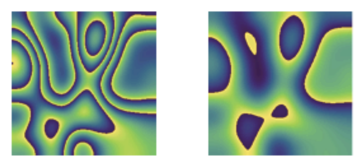


All 4 questions refer to the bottom pair of plots:

1) Which sample is farther off resonance, blue or black?

2) Do you have any way of knowing what reference frequency is actually being used?

3) If the 2 phase maps below, which were each acquired after letting the signal evolve for 2.46 ms, belong to the two samples that generated the signals in the plot, does the one on the left belong to the blue or the black signal?



4) If the sample were perfectly shimmed, what would the signal look like in the frequency domain? The time domain?